MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

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DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

DIGITAL NOTES of ELECTRICAL & ELECTRONICS ENGINEERING

For

B.Tech – II YEAR – I & II SEMESTER

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UNIT-I
ELECTRICAL CIRCUITS

Network theory is the study of solving the problems of electric circuits or electric networks. In this introductory chapter, let us first discuss the basic terminology of electric circuits and the types of network elements.

Basic Terminology

In Network Theory, we will frequently come across the following terms −

- Electric Circuit
- Electric Network
- Current
- Voltage
- Power

So, it is imperative that we gather some basic knowledge on these terms before proceeding further. Let’s start with Electric Circuit.

Electric Circuit

An electric circuit contains a closed path for providing a flow of electrons from a voltage source or current source. The elements present in an electric circuit will be in series connection, parallel connection, or in any combination of series and parallel connections.

Electric Network

An electric network need not contain a closed path for providing a flow of electrons from a voltage source or current source. Hence, we can conclude that "all electric circuits are electric networks" but the converse need not be true.

Current

The current **"I"** flowing through a conductor is nothing but the time rate of flow of charge. Mathematically, it can be written as

$$I = \frac{dQ}{dt}$$

Where,

- Q is the charge and its unit is Coloumb.

- t is the time and its unit is second.

As an analogy, electric current can be thought of as the flow of water through a pipe. Current is measured in terms of Ampere. In general, Electron current flows from negative terminal of source to positive terminal, whereas, Conventional current flows from positive terminal of source to negative terminal.

Electron current is obtained due to the movement of free electrons, whereas, Conventional current is obtained due to the movement of free positive charges. Both of these are called as electric current.

Voltage

The voltage "V" is nothing but an electromotive force that causes the charge (electrons) to flow. Mathematically, it can be written as

$$V = \frac{dW}{dQ}$$

Where,

- W is the potential energy and its unit is Joule.
- Q is the charge and its unit is Coloumb.

As an analogy, Voltage can be thought of as the pressure of water that causes the water to flow through a pipe. It is measured in terms of Volt.

Power

The power "P" is nothing but the time rate of flow of electrical energy. Mathematically, it can be written as

$$P = \frac{dW}{dt}$$

Where,

- W is the electrical energy and it is measured in terms of Joule.
- t is the time and it is measured in seconds.

We can re-write the above equation a

$$P = \frac{dW}{dt} = \frac{dW}{dQ} \times \frac{dQ}{dt} = VI$$

Therefore, power is nothing but the product of voltage V and current I. Its unit is Watt.

Types of Network Elements

We can classify the Network elements into various types based on some parameters. Following are the types of Network elements −

- Active Elements and Passive Elements
- Linear Elements and Non-linear Elements
- Bilateral Elements and Unilateral Elements

Active Elements and Passive Elements

We can classify the Network elements into either active or passive based on the ability of delivering power.

- Active Elements deliver power to other elements, which are present in an electric circuit. Sometimes, they may absorb the power like passive elements. That means active elements have the capability of both delivering and absorbing power. Examples: Voltage sources and current sources.
- Passive Elements can't deliver power (energy) to other elements, however they can absorb power. That means these elements either dissipate power in the form of heat or store energy in the form of either magnetic field or electric field. Examples: Resistors, Inductors, and capacitors.

Linear Elements and Non-Linear Elements

We can classify the network elements as linear or non-linear based on their characteristic to obey the property of linearity.

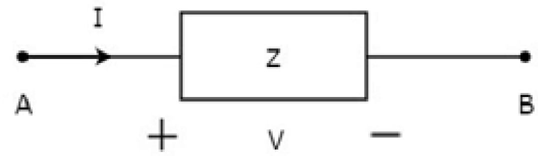
- Linear Elements are the elements that show a linear relationship between voltage and current. Examples: Resistors, Inductors, and capacitors.
- Non-Linear Elements are those that do not show a linear relation between voltage and current. Examples: Voltage sources and current sources.

Bilateral Elements and Unilateral Elements

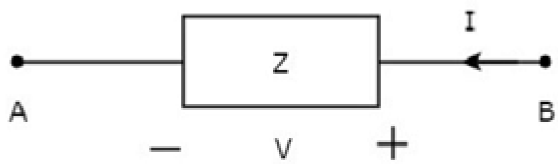
Network elements can also be classified as either bilateral or unilateral based on the direction of current flows through the network elements.

Bilateral Elements are the elements that allow the current in both directions and offer the same impedance in either direction of current flow. Examples: Resistors, Inductors and capacitors.

The concept of Bilateral elements is illustrated in the following figures.



In the above figure, the current (I) is flowing from terminals A to B through a passive element having impedance of $Z\,\Omega$. It is the ratio of voltage (V) across that element between terminals A & B and current (I).



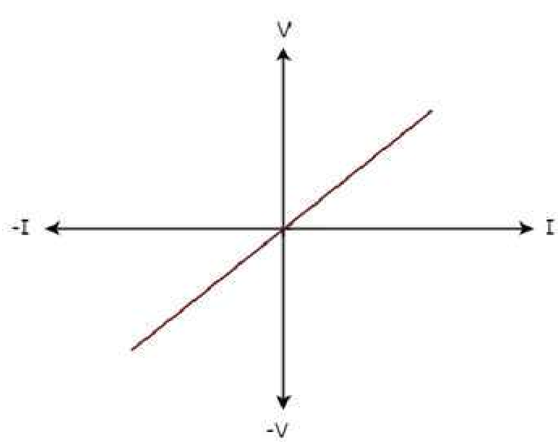
In the above figure, the current (I) is flowing from terminals B to A through a passive element having impedance of $Z\,\Omega$. That means the current (–I) is flowing from terminals A to B. In this case too, we will get the same impedance value, since both the current and voltage having negative signs with respect to terminals A & B.

Unilateral Elements are those that allow the current in only one direction. Hence, they offer different impedances in both directions.

We discussed the types of network elements in the previous chapter. Now, let us identify the nature of network elements from the V-I characteristics given in the following examples.

Example 1

The V-I characteristics of a network element is shown below.



Step 1 − Verifying the network element as linear or non-linear.

From the above figure, the V-I characteristics of a network element is a straight line passing through the origin. Hence, it is linear element.

Step 2 − Verifying the network element as active or passive.

The given V-I characteristics of a network element lies in the first and third quadrants.

- In the first quadrant, the values of both voltage (V) and current (I) are positive. So, the ratios of voltage (V) and current (I) gives positive impedance values.
- Similarly, in the third quadrant, the values of both voltage (V) and current (I) have negative values. So, the ratios of voltage (V) and current (I) produce positive impedance values.

Since, the given V-I characteristics offer positive impedance values, the network element is a Passive element.

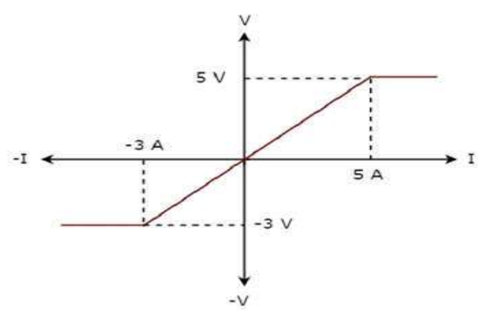
Step 3 − Verifying the network element as bilateral or unilateral.

For every point (I, V) on the characteristics, there exists a corresponding point (-I, -V) on the given characteristics. Hence, the network element is a Bilateral element.

Therefore, the given V-I characteristics show that the network element is a Linear, Passive, and Bilateral element.

Example 2

The V-I characteristics of a network element is shown below.



Step 1 − Verifying the network element as linear or non-linear.

From the above figure, the V-I characteristics of a network element is a straight line only between the points (-3A, -3V) and (5A, 5V). Beyond these points, the V-I characteristics are not following the linear relation. Hence, it is a Non-linear element.

Step 2 − Verifying the network element as active or passive.

The given V-I characteristics of a network element lies in the first and third quadrants. In these two quadrants, the ratios of voltage (V) and current (I) produce positive impedance values. Hence, the network element is a Passive element.

Step 3 − Verifying the network element as bilateral or unilateral.

Consider the point (5A, 5V) on the characteristics. The corresponding point (-5A, -3V) exists on the given characteristics instead of (-5A, -5V). Hence, the network element is a Unilateral element.

Therefore, the given V-I characteristics show that the network element is a Non-linear, Passive, and Unilateral element.

Network Theory - Active Elements

Active Elements are the network elements that deliver power to other elements present in an electric circuit. So, active elements are also called as sources of voltage or current type. We can classify these sources into the following two categories −

- Independent Sources
- Dependent Sources

Independent Sources

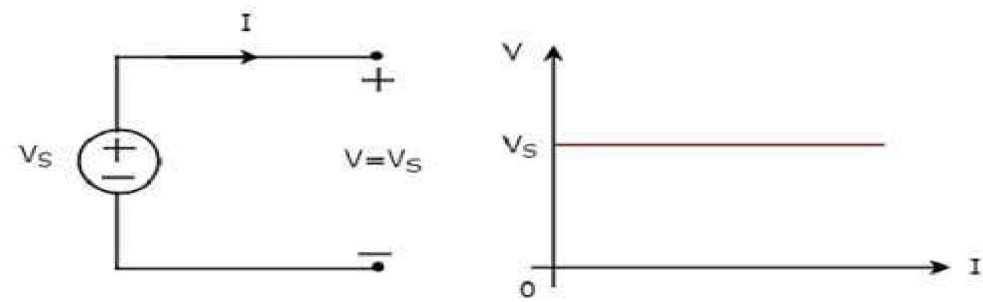
As the name suggests, independent sources produce fixed values of voltage or current and these are not dependent on any other parameter. Independent sources can be further divided into the following two categories −

- Independent Voltage Sources
- Independent Current Sources

Independent Voltage Sources

An independent voltage source produces a constant voltage across its two terminals. This voltage is independent of the amount of current that is flowing through the two terminals of voltage source.

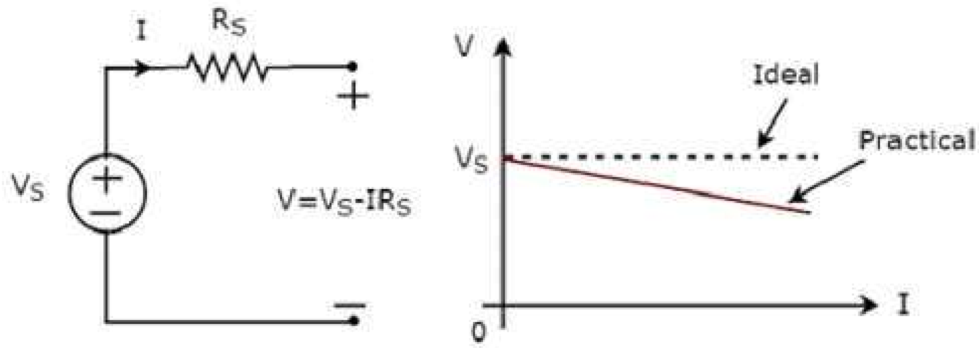
Independent ideal voltage source and its V-I characteristics are shown in the following figure.



The V-I characteristics of an independent ideal voltage source is a constant line, which is always equal to the source voltage (VS) irrespective of the current value (I). So, the internal resistance of an independent ideal voltage source is zero Ohms.

Hence, the independent ideal voltage sources do not exist practically, because there will be some internal resistance.

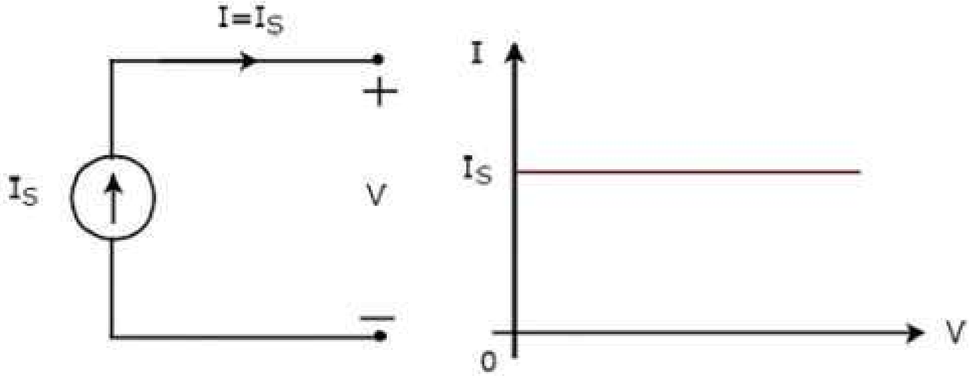
Independent practical voltage source and its V-I characteristics are shown in the following figure.



There is a deviation in the V-I characteristics of an independent practical voltage source from the V-I characteristics of an independent ideal voltage source. This is due to the voltage drop across the internal resistance (R_S) of an independent practical voltage source.

Independent Current Sources

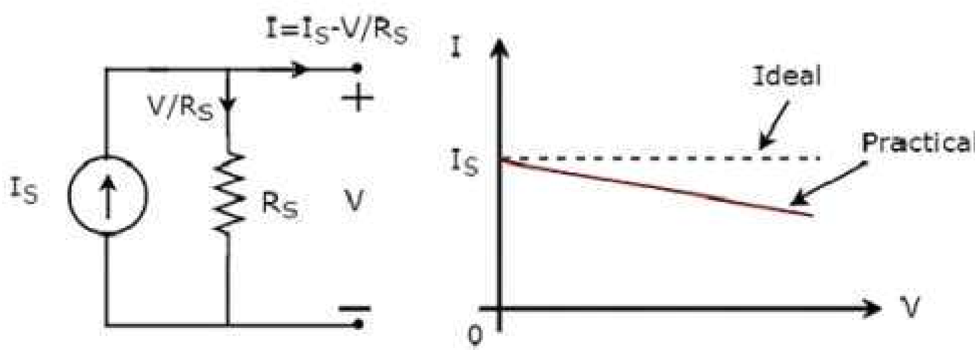
An independent current source produces a constant current. This current is independent of the voltage across its two terminals. Independent ideal current source and its V-I characteristics are shown in the following figure.



The V-I characteristics of an independent ideal current source is a constant line, which is always equal to the source current (I_S) irrespective of the voltage value (V). So, the internal resistance of an independent ideal current source is infinite ohms.

Hence, the independent ideal current sources do not exist practically, because there will be some internal resistance.

Independent practical current source and its V-I characteristics are shown in the following figure.



There is a deviation in the V-I characteristics of an independent practical current source from the V-I characteristics of an independent ideal current source. This is due to the amount of current flows through the internal shunt resistance (R_S) of an independent practical current source.

Dependent Sources

As the name suggests, dependent sources produce the amount of voltage or current that is dependent on some other voltage or current. Dependent sources are also called as controlled sources. Dependent sources can be further divided into the following two categories −

- Dependent Voltage Sources
- Dependent Current Sources

Dependent Voltage Sources

A dependent voltage source produces a voltage across its two terminals. The amount of this voltage is dependent on some other voltage or current. Hence, dependent voltage sources can be further classified into the following two categories −

- Voltage Dependent Voltage Source (VDVS)
- Current Dependent Voltage Source (CDVS)

Dependent voltage sources are represented with the signs '+' and '-' inside a diamond shape. The magnitude of the voltage source can be represented outside the diamond shape.

Dependent Current Sources

A dependent current source produces a current. The amount of this current is dependent on some other voltage or current. Hence, dependent current sources can be further classified into the following two categories −

- Voltage Dependent Current Source (VDCS)
- Current Dependent Current Source (CDCS)

Dependent current sources are represented with an arrow inside a diamond shape. The magnitude of the current source can be represented outside the diamond shape. We can observe these dependent or controlled sources in equivalent models of transistors.

Source Transformation Technique

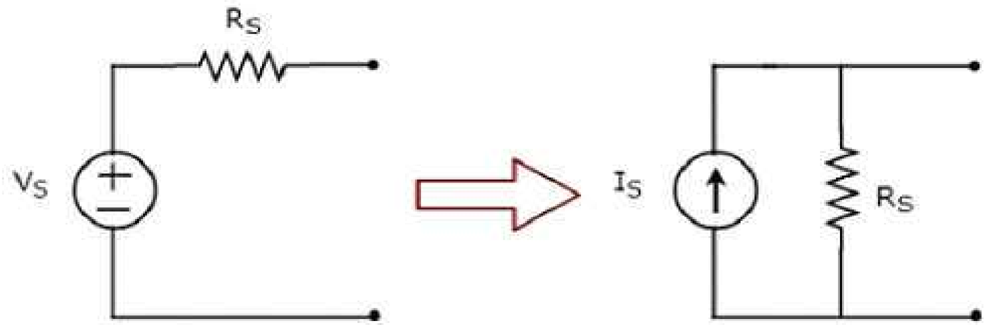
We know that there are two practical sources, namely, voltage source and current source. We can transform (convert) one source into the other based on the requirement, while solving network problems.

The technique of transforming one source into the other is called as source transformation technique. Following are the two possible source transformations −

- Practical voltage source into a practical current source
- Practical current source into a practical voltage source

Practical voltage source into a practical current source

The transformation of practical voltage source into a practical current source is shown in the following figure



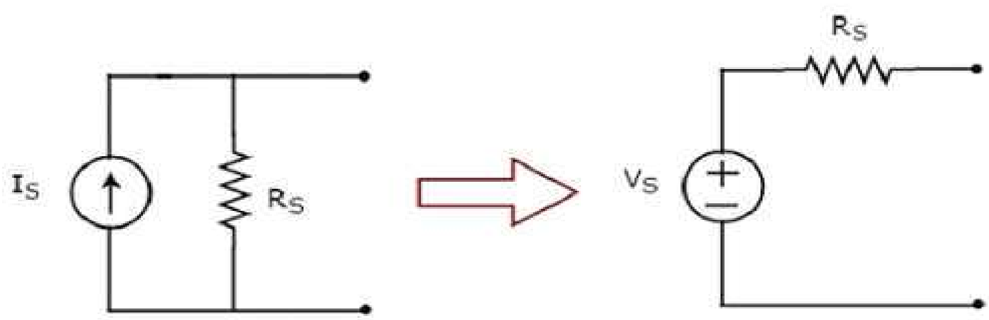
Practical voltage source consists of a voltage source (V_S) in series with a resistor (R_S). This can be converted into a practical current source as shown in the figure. It consists of a current source (I_S) in parallel with a resistor (R_S).

The value of IS will be equal to the ratio of V_S and R_S. Mathematically, it can be represented as

$$I_S = \frac{V_S}{R_S}$$

Practical current source into a practical voltage source

The transformation of practical current source into a practical voltage source is shown in the following figure.



Practical current source consists of a current source (I_S) in parallel with a resistor (R_S). This can be converted into a practical voltage source as shown in the figure. It consists of a voltage source (V_S) in series with a resistor (R_S).

The value of V_S will be equal to the product of I_S and R_S. Mathematically, it can be represented as

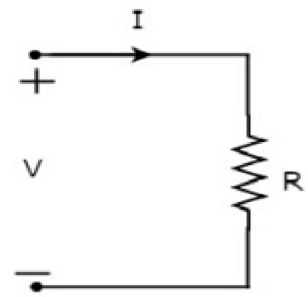
$$V_S = I_S R_S$$

In this chapter, we will discuss in detail about the passive elements such as Resistor, Inductor, and Capacitor. Let us start with Resistors.

Resistor

The main functionality of Resistor is either opposes or restricts the flow of electric current. Hence, the resistors are used in order to limit the amount of current flow and / or dividing (sharing) voltage.

Let the current flowing through the resistor is I amperes and the voltage across it is V volts. The symbol of resistor along with current, I and voltage, V are shown in the following figure.



According to Ohm's law, the voltage across resistor is the product of current flowing through it and the resistance of that resistor. Mathematically, it can be represented as

$$V = IR \qquad \text{Equation 1}$$

$$\Rightarrow I = \frac{V}{R} \qquad \text{Equation 2}$$

Where, **R** is the resistance of a resistor.

From Equation 2, we can conclude that the current flowing through the resistor is directly proportional to the applied voltage across resistor and inversely proportional to the resistance of resistor.

Power in an electric circuit element can be represented as

$$P = VI \qquad \text{Equation 3}$$

Substitute, Equation 1 in Equation 3.

$$P = (IR)I$$

$$\Rightarrow P = I^2R \qquad \text{Equation 4}$$

Substitute, Equation 2 in Equation 3.

$$P = V\left(\frac{V}{R}\right)$$

$$\Rightarrow P = \frac{V^2}{R} \qquad \text{Equation 5}$$

So, we can calculate the amount of power dissipated in the resistor by using one of the formulae mentioned in Equations 3 to 5.

Inductor

In general, inductors will have number of turns. Hence, they produce magnetic flux when current flows through it. So, the amount of total magnetic flux produced by an inductor depends on the current, I flowing through it and they have linear relationship.

Mathematically, it can be written as

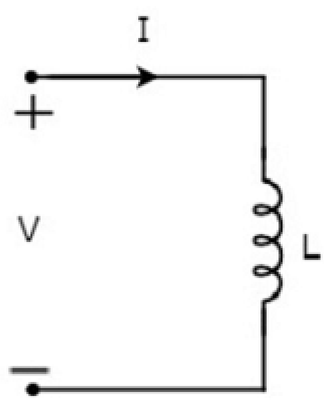
$$\Psi \, \alpha \, I$$

$$\Rightarrow \Psi = LI$$

Where,

- $\boldsymbol{\Psi}$ is the total magnetic flux
- $\boldsymbol{L}$ is the inductance of an inductor

Let the current flowing through the inductor is I amperes and the voltage across it is V volts. The symbol of inductor along with current I and voltage V are shown in the following figure.



According to Faraday's law, the voltage across the inductor can be written as

$$V = \frac{d\Psi}{dt}$$

Substitute $\Psi = LI$ in the above equation.

$$V = \frac{d(LI)}{dt}$$

$$\Rightarrow V = L\frac{dI}{dt}$$

$$\Rightarrow I = \frac{1}{L}\int V dt$$

From the above equations, we can conclude that there exists a linear relationship between voltage across inductor and current flowing through it.

We know that power in an electric circuit element can be represented as

$$P = VI$$

Substitute $V = L\frac{dI}{dt}$ in the above equation.

$$P = (L\frac{dI}{dt})I$$

$$\Rightarrow P = LI\frac{dI}{dt}$$

By integrating the above equation, we will get the energy stored in an inductor as

$$W = \frac{1}{2}LI^2$$

So, the inductor stores the energy in the form of magnetic field.

Capacitor

In general, a capacitor has two conducting plates, separated by a dielectric medium. If positive voltage is applied across the capacitor, then it stores positive charge. Similarly, if negative voltage is applied across the capacitor, then it stores negative charge.

So, the amount of charge stored in the capacitor depends on the applied voltage **V** across it and they have linear relationship. Mathematically, it can be written as

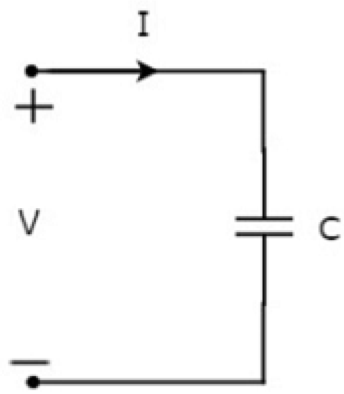
$$Q \alpha V$$

$$\Rightarrow Q = CV$$

Where,

- ***Q*** is the charge stored in the capacitor.
- ***C*** is the capacitance of a capacitor.

Let the current flowing through the capacitor is *I* amperes and the voltage across it is *V* volts. The symbol of capacitor along with current *I* and voltage *V* are shown in the following figure.



We know that the **current** is nothing but the **time rate of flow of charge**. Mathematically, it can be represented as

$$I = \frac{dQ}{dt}$$

Substitute $Q = CV$ in the above equation.

$$I = \frac{d(CV)}{dt}$$

$$\Rightarrow I = C\frac{dV}{dt}$$

$$\Rightarrow V = \frac{1}{C}\int I dt$$

From the above equations, we can conclude that there exists a linear relationship between voltage across capacitor and current flowing through it.

We know that power in an electric circuit element can be represented as

$$P = VI$$

Substitute $I = C\frac{dV}{dt}$ in the above equation.

$$P = V(C\frac{dV}{dt})$$

$$\Rightarrow P = CV\frac{dV}{dt}$$

By integrating the above equation, we will get the **energy** stored in the capacitor as

$$W = \frac{1}{2}CV^2$$

So, the capacitor stores the energy in the form of electric field.

Network Theory - Kirchhoff's Laws

Network elements can be either of active or passive type. Any electrical circuit or network contains one of these two types of network elements or a combination of both.

Now, let us discuss about the following two laws, which are popularly known as Kirchhoff's laws.

- Kirchhoff's Current Law
- Kirchhoff's Voltage Law

Kirchhoff's Current Law

Kirchhoff's Current Law (KCL) states that the algebraic sum of currents leaving (or entering) a node is equal to zero.

A Node is a point where two or more circuit elements are connected to it. If only two circuit elements are connected to a node, then it is said to be simple node. If three or more circuit elements are connected to a node, then it is said to be Principal Node.

Mathematically, KCL can be represented as

$$\sum_{m=1}^{M} I_m = 0$$

Where,

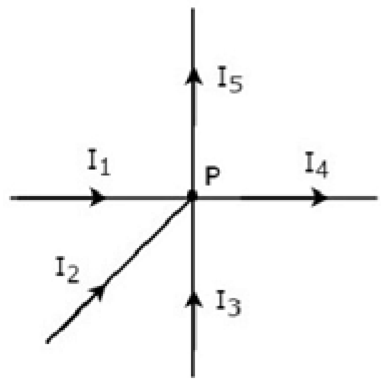
- I_m is the m^{th} branch current leaving the node.

- ***M*** is the number of branches that are connected to a node.

The above statement of KCL can also be expressed as "the algebraic sum of currents entering a node is equal to the algebraic sum of currents leaving a node". Let us verify this statement through the following example.

Example

Write KCL equation at node P of the following figure.



- In the above figure, the branch currents I_1, I_2 and I_3 areentering at node P. So, consider negative signs for these three currents.
- In the above figure, the branch currents I_4 and I_5 areleaving from node P. So, consider positive signs for these two currents.

The KCL equation at node P will be

$$-I_1 - I_2 - I_3 + I_4 + I_5 = 0$$

$$\Rightarrow I_1 + I_2 + I_3 = I_4 + I_5$$

In the above equation, the left-hand side represents the sum of entering currents, whereas the right-hand side represents the sum of leaving currents.

In this tutorial, we will consider positive sign when the current leaves a node and negative sign when it enters a node. Similarly, you can consider negative sign when the current leaves a node and positive sign when it enters a node. In both cases, the result will be same.

Note − KCL is independent of the nature of network elements that are connected to a node.

Kirchhoff's Voltage Law

Kirchhoff's Voltage Law (KVL) states that the algebraic sum of voltages around a loop or mesh is equal to zero.

A Loop is a path that terminates at the same node where it started from. In contrast, a Mesh is a loop that doesn't contain any other loops inside it.

Mathematically, KVL can be represented as

$$\sum_{n=1}^{N} V_n = 0$$

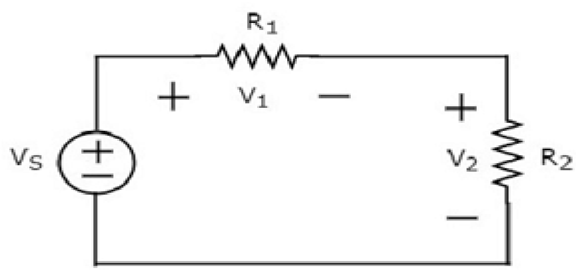
Where,

- $\mathbf{V_n}$ is the n^{th} element's voltage in a loop (mesh).
- **N** is the number of network elements in the loop (mesh).

The above statement of KVL can also be expressed as "the algebraic sum of voltage sources is equal to the algebraic sum of voltage drops that are present in a loop." Let us verify this statement with the help of the following example.

Example

Write KVL equation around the loop of the following circuit.



The above circuit diagram consists of a voltage source, V_S in series with two resistors R_1 and R_2. The voltage drops across the resistors R_1 and R_2 are V_1 and V_2 respectively.

Apply KVL around the loop.

$$V_S - V_1 - V_2 = 0$$

$$\Rightarrow V_S = V_1 + V_2$$

In the above equation, the left-hand side term represents single voltage source VS. Whereas, the right-hand side represents the sum of voltage drops. In this example, we considered only one voltage source. That's why the left-hand side contains only one term. If we consider multiple voltage sources, then the left side contains sum of voltage sources.

In this tutorial, we consider the sign of each element's voltage as the polarity of the second terminal that is present while travelling around the loop. Similarly, you can consider the sign of each voltage as the polarity of the first terminal that is present while travelling around the loop. In both cases, the result will be same.

Note − KVL is independent of the nature of network elements that are present in a loop.

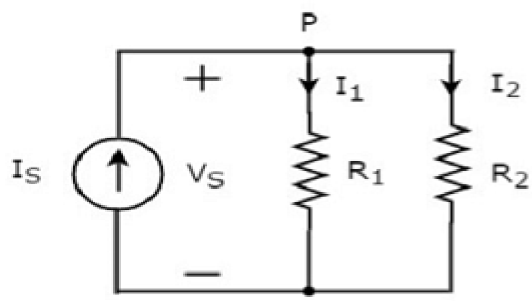
In this chapter, let us discuss about the following two division principles of electrical quantities.

- Current Division Principle
- Voltage Division Principle

Current Division Principle

When two or more passive elements are connected in parallel, the amount of current that flows through each element gets divided(shared) among themselves from the current that is entering the node.

Consider the following circuit diagram.



The above circuit diagram consists of an input current source $\mathbf{I_S}$ in parallel with two resistors $\mathbf{R_1}$ and $\mathbf{R_2}$. The voltage across each element is $\mathbf{V_S}$. The currents flowing through the resistors $\mathbf{R_1}$ and$\mathbf{R_2}$ are $\mathbf{I_1}$ and $\mathbf{I_2}$ respectively.

The KCL equation at node P will be

$$I_S = I_1 + I_2$$

- Substitute $I_1 = \frac{V_S}{R_1}$ and $I_2 = \frac{V_S}{R_2}$ in the above equation.

$$I_S = \frac{V_S}{R_1} + \frac{V_S}{R_2} = V_S\left(\frac{R_2 + R_1}{R_1 R_2}\right)$$

$$\Rightarrow V_S = I_S\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

- Substitute the value of V_S in $I_1 = \frac{V_S}{R_1}$.

$$I_1 = \frac{I_S}{R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$\Rightarrow I_1 = I_S\left(\frac{R_2}{R_1 + R_2}\right)$$

- Substitute the value of V_S in $I_2 = \frac{V_S}{R_2}$.

$$I_2 = \frac{I_S}{R_2}\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$\Rightarrow I_2 = I_S\left(\frac{R_1}{R_1 + R_2}\right)$$

From equations of I_1 and I_2, we can generalize that the current flowing through any passive element can be found by using the following formula.

$$I_N = I_S\left(\frac{Z_1 \| Z_2 \| \ldots \| Z_{N-1}}{Z_1 + Z_2 + \ldots + Z_N}\right)$$

This is known as current division principle and it is applicable, when two or more passive elements are connected in parallel and only one current enters the node.

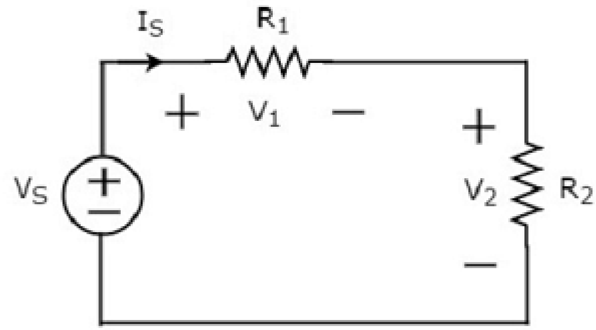
Where,

- I_N is the current flowing through the passive element of N^{th} branch.
- I_S is the input current, which enters the node.
- $Z_1, Z_2, \ldots, Z_N$ are the impedances of 1^{st} branch, 2^{nd} branch, …, N^{th} branch respectively.

Voltage Division Principle

When two or more passive elements are connected in series, the amount of voltage present across each element gets divided(shared) among themselves from the voltage that is available across that entire combination.

Consider the following circuit diagram.



The above circuit diagram consists of a voltage source, V_S in series with two resistors R_1 and R_2. The current flowing through these elements is I_S. The voltage drops across the resistors $R_1$and R_2 are V_1 and V_2 respectively.

The KVL equation around the loop will be

$$V_S = V_1 + V_2$$

- Substitute $V_1 = I_S R_1$ and $V_2 = I_S R_2$ in the above equation

$$V_S = I_S R_1 + I_S R_2 = I_S (R_1 + R_2)$$

$$I_S = \frac{V_S}{R_1 + R_2}$$

- Substitute the value of I_S in $V_1 = I_S R_1$.

$$V_1 = (\frac{V_S}{R_1 + R_2})R_1$$

$$\Rightarrow V_1 = V_S(\frac{R_1}{R_1 + R_2})$$

- Substitute the value of I_S in $V_2 = I_S R_2$.

$$V_2 = (\frac{V_S}{R_1 + R_2})R_2$$

$$\Rightarrow V_2 = V_S(\frac{R_2}{R_1 + R_2})$$

From equations of V_1 and V_2, we can generalize that the voltage across any passive element can be found by using the following formula.

$$V_N = V_S\left(\frac{Z_N}{Z_1 + Z_2 + \ldots + Z_N}\right)$$

This is known as voltage division principle and it is applicable, when two or more passive elements are connected in series and only one voltage available across the entire combination.

Where,

- V_N is the voltage across N^{th} passive element.
- V_S is the input voltage, which is present across the entire combination of series passive elements.
- $Z_1, Z_2, \ldots, Z_3$ are the impedances of 1^{st} passive element, 2^{nd} passive element, …, N^{th} passive element respectively.

There are two basic methods that are used for solving any electrical network: Nodal analysis and Mesh analysis. In this chapter, let us discuss about the Nodal analysis method.

In Nodal analysis, we will consider the node voltages with respect to Ground. Hence, Nodal analysis is also called as Node-voltage method.

Procedure of Nodal Analysis

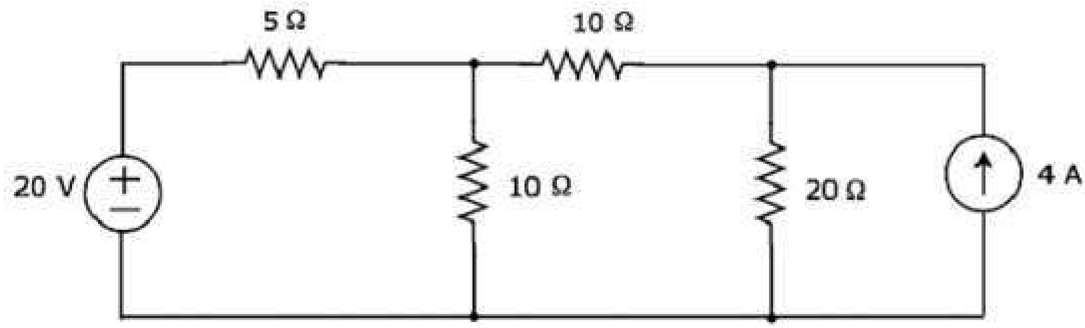
Follow these steps while solving any electrical network or circuit using Nodal analysis.

- **Step 1** − Identify the principal nodes and choose one of them as reference node. We will treat that reference node as the Ground.
- **Step 2** − Label the node voltages with respect to Ground from all the principal nodes except the reference node.
- **Step 3** − Write nodal equations at all the principal nodes except the reference node. Nodal equation is obtained by applying KCL first and then Ohm's law.
- **Step 4** − Solve the nodal equations obtained in Step 3 in order to get the node voltages.

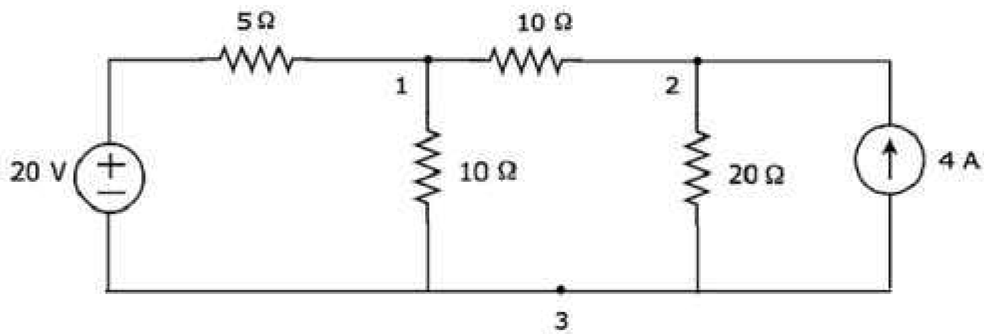
Now, we can find the current flowing through any element and the voltage across any element that is present in the given network by using node voltages.

Example

Find the current flowing through 20 Ω resistor of the following circuit using Nodal analysis.

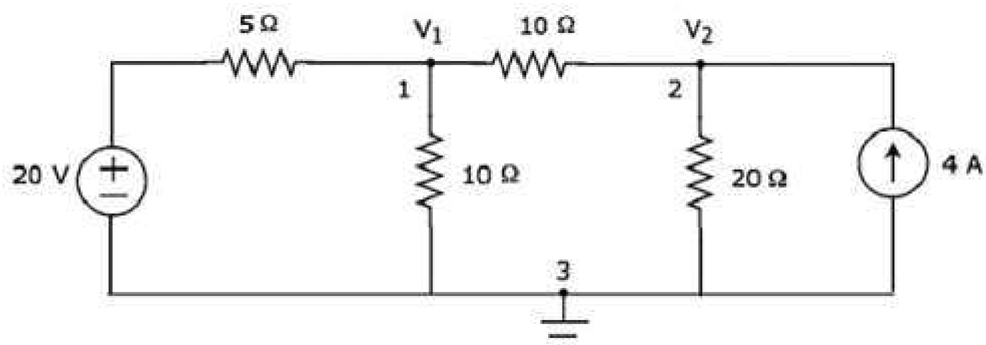


Step 1 − There are three principle nodes in the above circuit. Those are labeled as 1, 2, and 3 in the following figure.



In the above figure, consider node 3 as reference node (Ground).

Step 2 − The node voltages, V_1 and V_2, are labeled in the following figure.



In the above figure, V_1 is the voltage from node 1 with respect to ground and V_2 is the voltage from node 2 with respect to ground.

Step 3 − In this case, we will get two nodal equations, since there are two principal nodes, 1 and 2, other than Ground. When we write the nodal equations at a node, assume all the currents are leaving from the node for which the direction of current is not mentioned and that node's voltage as greater than other node voltages in the circuit.

The nodal equation at node 1 is

$$\frac{V_1 - 20}{5} + \frac{V_1}{10} + \frac{V_1 - V_2}{10} = 0$$

$$\Rightarrow \frac{2V_1 - 40 + V_1 + V_1 - V_2}{10} = 0$$

$$\Rightarrow 4V_1 - 40 - V_2 = 0$$

$$\Rightarrow V_2 = 4V_1 - 40$$

Equation 1

The **nodal equation** at node 2 is

$$-4 + \frac{V_2}{20} + \frac{V_2 - V_1}{10} = 0$$

$$\Rightarrow \frac{-80 + V_2 + 2V_2 - 2V_2}{20} = 0$$

$$\Rightarrow 3V_2 - 2V_1 = 80$$

Equation 2

Step 4 − Finding node voltages, V_1 and V_2 by solving Equation 1 and Equation 2.

Substitute Equation 1 in Equation 2.

$$3(4V_1 - 40) - 2V_1 = 80$$

$$\Rightarrow 12V_1 - 120 - 2V_1 = 80$$

$$\Rightarrow 10V_1 = 200$$

$$\Rightarrow V_1 = 20V$$

Substitute V_1 = 20 V in Equation1.

$$V_2 = 4(20) - 40$$

$$\Rightarrow V_2 = 40V$$

So, we got the node voltages V_1 and V_2 as 20 V and 40 Vrespectively.

Step 5 − The voltage across 20 Ω resistor is nothing but the node voltage V_2 and it is equal to 40 V. Now, we can find the current flowing through 20 Ω resistor by using Ohm's law.

$$I_{20\Omega} = \frac{V_2}{R}$$

Substitute the values of V_2 and R in the above equation.

$$I_{20\Omega} = \frac{40}{20}$$

$$\Rightarrow I_{20\Omega} = 2A$$

Therefore, the current flowing through 20 Ω resistor of given circuit is **2 A**.

Note − From the above example, we can conclude that we have to solve 'n' nodal equations, if the electric circuit has 'n' principal nodes (except the reference node). Therefore, we can choose Nodal analysis when the number of principal nodes (except reference node) is less than the number of meshes of any electrical circuit.

Mesh Analysis

In Mesh analysis, we will consider the currents flowing through each mesh. Hence, Mesh analysis is also called as Mesh-current method.

A branch is a path that joins two nodes and it contains a circuit element. If a branch belongs to only one mesh, then the branch current will be equal to mesh current.

If a branch is common to two meshes, then the branch current will be equal to the sum (or difference) of two mesh currents, when they are in same (or opposite) direction.

Procedure of Mesh Analysis

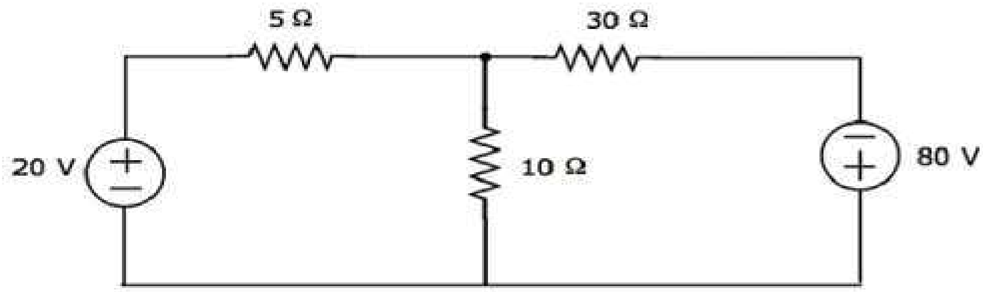
Follow these steps while solving any electrical network or circuit using Mesh analysis.

- **Step 1** − Identify the meshes and label the mesh currents in either clockwise or anti-clockwise direction.
- **Step 2** − Observe the amount of current that flows through each element in terms of mesh currents.
- **Step 3** − Write mesh equations to all meshes. Mesh equation is obtained by applying KVL first and then Ohm's law.
- **Step 4** − Solve the mesh equations obtained in Step 3 in order to get the mesh currents.

Now, we can find the current flowing through any element and the voltage across any element that is present in the given network by using mesh currents.

Example

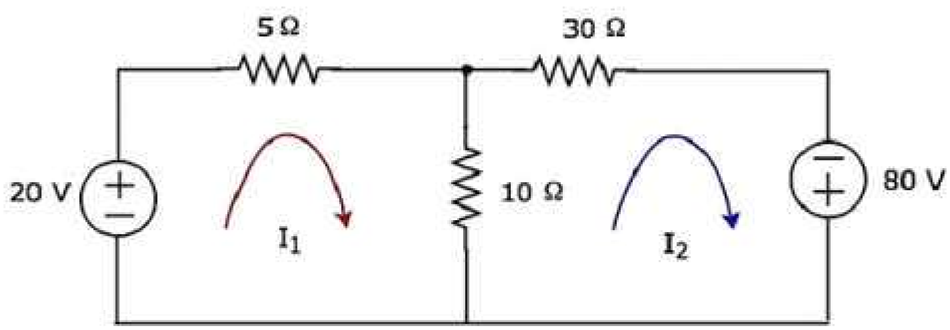
Find the voltage across 30 Ω resistor using Mesh analysis.



Step 1 − There are two meshes in the above circuit. The mesh currents I_1 and I_2 are considered in clockwise direction. These mesh currents are shown in the following figure.

Step 2 − The mesh current I_1 flows through 20 V voltage source and 5 Ω resistor. Similarly, the mesh current I_2 flows through 30 Ω resistor and -80 V voltage source. But, the difference of two mesh currents, I_1 and I_2, flows through 10 Ω resistor, since it is the common branch of two meshes.

Step 3 − In this case, we will get two mesh equations since there are two meshes in the given circuit. When we write the mesh equations, assume the mesh current of that particular mesh as greater than all other mesh currents of the circuit. The mesh equation of first mesh is



$$20 - 5I_1 - 10(I_1 - I_2) = 0$$

$$\Rightarrow 20 - 15I_1 + 10I_2 = 0$$

$$\Rightarrow 10I_2 = 15I_1 - 20$$

Divide the above equation with 5.

$$2I_2 = 3I_1 - 4$$

Multiply the above equation with 2.

$$4I_2 = 6I_1 - 8$$ Equation 1

The **mesh equation** of second mesh is

$$-10(I_2 - I_1) - 30I_2 + 80 = 0$$

Divide the above equation with 10.

$$-(I_2 - I_1) - 3I_2 + 8 = 0$$

$$\Rightarrow -4I_2 + I_1 + 8 = 0$$

$$4I_2 = I_1 + 8$$ Equation 2

Step 4 − Finding mesh currents I_1 and I_2 by solving Equation 1 and Equation 2.

The left-hand side terms of Equation 1 and Equation 2 are the same. Hence, equate the right-hand side terms of Equation 1 and Equation 2 in order find the value of I_1.

$$6I_1 - 8 = I_1 + 8$$

$$\Rightarrow 5I_1 = 16$$

$$\Rightarrow I_1 = \frac{16}{5}A$$

Substitute I_1 value in Equation 2.

$$4I_2 = \frac{16}{5} + 8$$

$$\Rightarrow 4I_2 = \frac{56}{5}$$

$$\Rightarrow I_2 = \frac{14}{5}A$$

So, we got the mesh currents I_1 and I_2 as $\frac{16}{5}$ A and $\frac{14}{5}$ A respectively.

Step 5 – The current flowing through 30 Ω resistor is nothing but the mesh current I_2 and it is equal to $\frac{14}{5}$ A. Now, we can find the voltage across 30 Ω resistor by using Ohm's law.

$$V_{30\Omega} = I_2R$$

Substitute the values of I_2 and R in the above equation.

$$V_{30\Omega} = (\frac{14}{5})30$$

$$\Rightarrow V_{30\Omega} = 84V$$

Therefore, the voltage across 30 Ω resistor of the given circuit is84 V.

Note 1 − From the above example, we can conclude that we have to solve 'm' mesh equations, if the electric circuit is having 'm' meshes. That's why we can choose Mesh analysis when the number of meshes is less than the number of principal nodes (except the reference node) of any electrical circuit.

Note 2 − We can choose either Nodal analysis or Mesh analysis, when the number of meshes is equal to the number of principal nodes (except the reference node) in any electric circuit.

If a circuit consists of two or more similar passive elements and are connected in exclusively of series type or parallel type, then we can replace them with a single equivalent passive element. Hence, this circuit is called as an equivalent circuit.

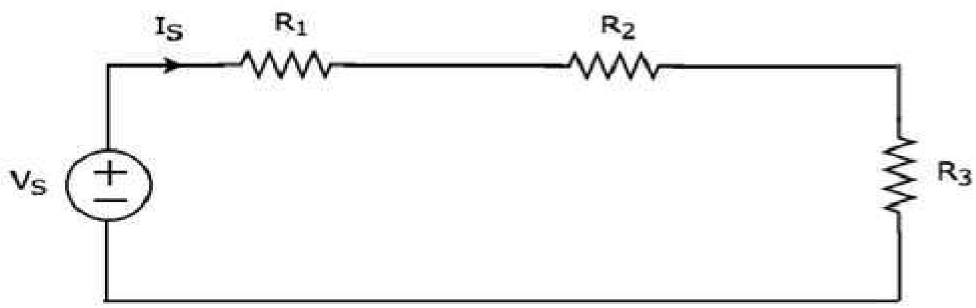
In this chapter, let us discuss about the following two equivalent circuits.

- Series Equivalent Circuit
- Parallel Equivalent Circuit

Series Equivalent Circuit

If similar passive elements are connected in series, then the same current will flow through all these elements. But, the voltage gets divided across each element.

Consider the following circuit diagram.



It has a single voltage source (V_S) and three resistors having resistances of R_1, R_2 and R_3. All these elements are connected in series. The current IS flows through all these elements.

The above circuit has only one mesh. The KVL equation around this mesh is

$$V_S = V_1 + V_2 + V_3$$

Substitute $V_1 = I_S R_1$, $V_2 = I_S R_2$ and $V_3 = I_S R_3$ in the above equation.

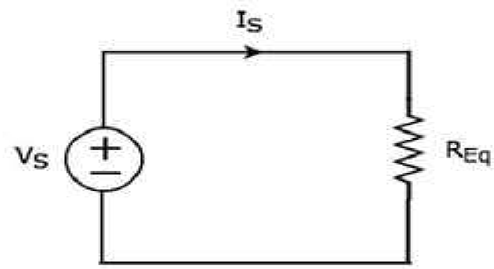
$$V_S = I_S R_1 + I_S R_2 + I_S R_3$$

$$\Rightarrow V_S = I_S (R_1 + R_2 + R_3)$$

The above equation is in the form of $V_S = I_S R_{Eq}$ where,

$$R_{Eq} = R_1 + R_2 + R_3$$

The equivalent circuit diagram of the given circuit is shown in the following figure.



That means, if multiple resistors are connected in series, then we can replace them with an equivalent resistor. The resistance of this equivalent resistor is equal to sum of the resistances of all those multiple resistors.

Note 1 − If 'N' inductors having inductances of L_1, L_2, ..., L_N are connected in series, then the equivalent inductance will be

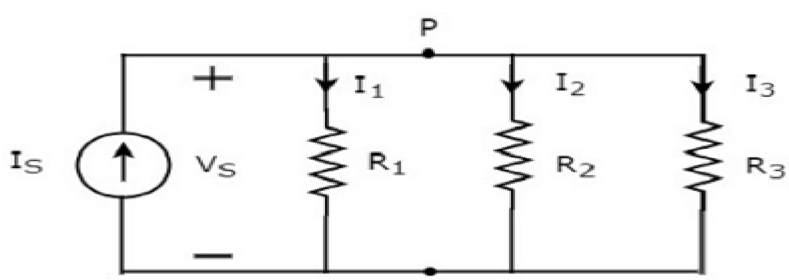
$$L_{Eq} = L_1 + L_2 + \ldots + L_N$$

Note 2 − If 'N' capacitors having capacitances of C_1, C_2, ..., $C_N$are connected in series, then the equivalent capacitance will be

$$\frac{1}{C_{Eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}$$

Parallel Equivalent Circuit

If similar passive elements are connected in parallel, then the same voltage will be maintained across each element. But, the current flowing through each element gets divided. Consider the following circuit diagram.



It has a single current source (I_S) and three resistors having resistances of R_1, R_2, and R_3. All these elements are connected in parallel. The voltage (V_S) is available across all these elements. The above circuit has only one principal node (P) except the Ground node. The KCL equation at this principal node (P) is

$$I_S = I_1 + I_2 + I_3$$

Substitute $I_1 = \frac{V_S}{R_1}$, $I_2 = \frac{V_S}{R_2}$ and $I_3 = \frac{V_S}{R_3}$ in the above equation.

$$I_S = \frac{V_S}{R_1} + \frac{V_S}{R_2} + \frac{V_S}{R_3}$$

$$\Rightarrow I_S = V_S\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

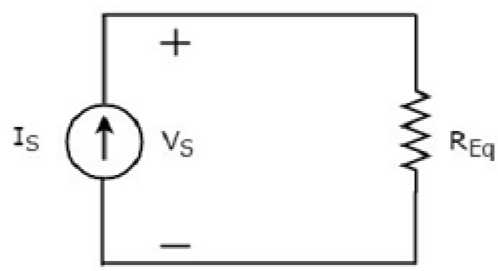
$$\Rightarrow V_S = I_S\left[\frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}\right]$$

The above equation is in the form of $V_S = I_S R_{Eq}$ where,

$$R_{Eq} = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

$$\frac{1}{R_{Eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The equivalent circuit diagram of the given circuit is shown in the following figure.



That means, if multiple resistors are connected in parallel, then we can replace them with an equivalent resistor. The resistance of this equivalent resistor is equal to the reciprocal of sum of reciprocal of each resistance of all those multiple resistors.

Note 1 − If 'N' inductors having inductances of L_1, L_2, ..., L_N are connected in parallel, then the equivalent inductance will be

$$\frac{1}{L_{Eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}$$

Note 2 − If 'N' capacitors having capacitances of C_1, C_2, ..., $C_N$are connected in parallel, then the equivalent capacitance will be

$$C_{Eq} = C_1 + C_2 + \ldots + C_N$$

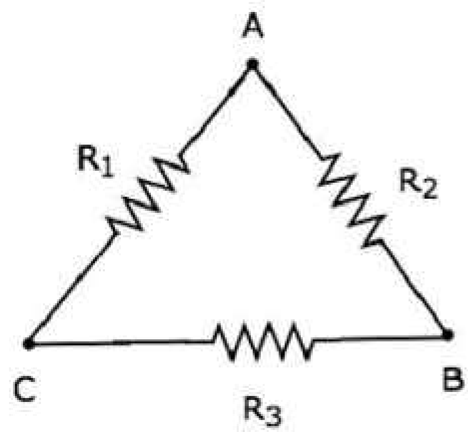
Delta to Star Conversion

In the previous chapter, we discussed an example problem related equivalent resistance. There, we calculated the equivalent resistance between the terminals A & B of the given electrical network easily. Because, in every step, we got the combination of resistors that are connected in either series form or parallel form.

However, in some situations, it is difficult to simplify the network by following the previous approach. For example, the resistors connected in either delta (δ) form or star form. In such situations, we have to convert the network of one form to the other in order to simplify it further by using series combination or parallel combination. In this chapter, let us discuss about the Delta to Star Conversion.

Delta Network

Consider the following delta network as shown in the following figure.



The following equations represent the equivalent resistance between two terminals of delta network, when the third terminal is kept open.

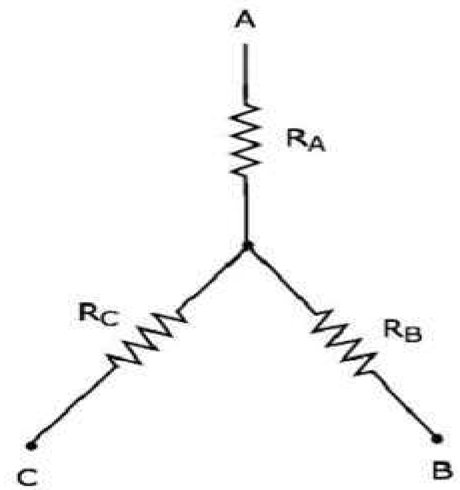
$$R_{AB} = \frac{(R_1 + R_3)R_2}{R_1 + R_2 + R_3}$$

$$R_{BC} = \frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3}$$

$$R_{CA} = \frac{(R_2 + R_3)R_1}{R_1 + R_2 + R_3}$$

Star Network

The following figure shows the equivalent star network corresponding to the above delta network.



The following equations represent the equivalent resistance between two terminals of star network, when the third terminal is kept open.

$$R_{AB} = R_A + R_B$$

$$R_{BC} = R_B + R_C$$

$$R_{CA} = R_C + R_A$$

Star Network Resistances in terms of Delta Network Resistances

We will get the following equations by equating the right-hand side terms of the above equations for which the left-hand side terms are same.

$$R_A + R_B = \frac{(R_1+R_3)R_2}{R_1+R_2+R_3}$$ Equation

1

$$R_B + R_C = \frac{(R_1+R_2)R_3}{R_1+R_2+R_3}$$ Equation

2

$$R_C + R_A = \frac{(R_2+R_3)R_1}{R_1+R_2+R_3}$$ Equation

3

By adding the above three equations, we will get

$$2(R_A + R_B + R_C) = \frac{2(R_1R_2 + R_2R_3 + R_3R_1)}{R_1 + R_2 + R_3}$$

$$\Rightarrow R_A + R_B + R_C = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1+R_2+R_3}$$ Equation

4

Subtract Equation 2 from Equation 4.

$$R_A + R_B + R_C - (R_B + R_C) = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1+R_2+R_3} - \frac{(R_1+R_2)R_3}{R_1+R_2+R_3}$$

Subtract Equation 2 from Equation 4.

$$R_A + R_B + R_C - (R_B + R_C) = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1+R_2+R_3} - \frac{(R_1+R_2)R_3}{R_1+R_2+R_3}$$

$$R_A = \frac{R_1R_2}{R_1 + R_2 + R_3}$$

By subtracting Equation 3 from Equation 4, we will get

$$R_B = \frac{R_2R_3}{R_1 + R_2 + R_3}$$

By subtracting Equation 1 from Equation 4, we will get

$$R_C = \frac{R_3R_1}{R_1 + R_2 + R_3}$$

By using the above relations, we can find the resistances of star network from the resistances of delta network. In this way, we can convert a delta network into a star network.

Star to Delta Conversion

In the previous chapter, we discussed about the conversion of delta network into an equivalent star network. Now, let us discuss about the conversion of star network into an equivalent delta network. This conversion is called as Star to Delta Conversion.

In the previous chapter, we got the resistances of star network from delta network as

$$R_A = \frac{R_1 R_2}{R_1+R_2+R_3} \qquad \text{Equation 1}$$

$$R_B = \frac{R_2 R_3}{R_1+R_2+R_3} \qquad \text{Equation 2}$$

$$R_C = \frac{R_3 R_1}{R_1+R_2+R_3} \qquad \text{Equation 3}$$

Delta Network Resistances in terms of Star Network Resistances

Let us manipulate the above equations in order to get the resistances of delta network in terms of resistances of star network.

- Multiply each set of two equations and then add.

$$R_AR_B + R_BR_C + R_CR_A = \frac{R_1R_2^2R_3 + R_2R_3^2R_1 + R_3R_1^2R_2}{(R_1 + R_2 + R_3)^2}$$

$$\Rightarrow R_AR_B + R_BR_C + R_CR_A = \frac{R_1R_2R_3(R_1 + R_2 + R_3)}{(R_1 + R_2 + R_3)^2}$$

$$\Rightarrow R_AR_B + R_BR_C + R_CR_A = \frac{R_1R_2R_3}{R_1+R_2+R_3}$$

Equation 4

- By dividing Equation 4 with Equation 2, we will get

$$\frac{R_AR_B + R_BR_C + R_CR_A}{R_B} = R_1$$

$$\Rightarrow R_1 = R_C + R_A + \frac{R_CR_A}{R_B}$$

- By dividing Equation 4 with Equation 3, we will get

$$R_2 = R_A + R_B + \frac{R_AR_B}{R_C}$$

- By dividing Equation 4 with Equation 1, we will get

$$R_3 = R_B + R_C + \frac{R_BR_C}{R_A}$$

By using the above relations, we can find the resistances of delta network from the resistances of star network. In this way, we can convert star network into delta network.

Electrical Measuring Instruments

Basically there are three types of measuring instruments and they are

1. Electrical measuring instruments
2. Mechanical measuring instruments.
3. Electronic measuring instruments.

Here we are interested in electrical measuring instruments so we will discuss about them in detail. Electrical instruments measure the various electrical quantities like electrical power factor, power, voltage and current etc. All analog electrical instruments use mechanical system for the measurement of various electrical quantities but as we know the all mechanical system has some inertia therefore electrical instruments have a limited time response.

Now there are various ways of classifying the instruments. On broad scale we can categorize them as:

Absolute Measuring Instruments

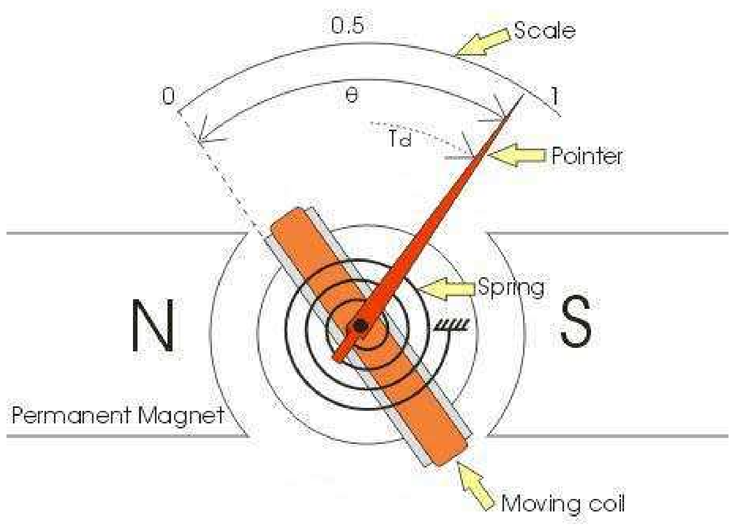
These instruments give output in terms of physical constant of the instruments. For example Rayleigh's current balance and Tangent galvanometer are absolute instruments.

Secondary Measuring Instruments

These instruments are constructed with the help of absolute instruments. Secondary instruments are calibrated by comparison with an absolute instruments. These are more frequently used in measurement of the quantities as compared to absolute instruments, as working with absolute instruments is time consuming. Another way of classifying the electrical measuring instruments depends on the way they produce the result of measurements. On this basis they can be of two types:

Deflection Type Instruments

In these types of instruments, pointer of the electrical measuring instrument deflects to measure the quantity. The value of the quantity can be measured by measuring the net deflection of the pointer from its initial position. In order to understand these types of instruments let us take an example of deflection type permanent magnet moving coil ammeter which is shown below:



The diagram shown above has two permanent magnets which are called the stationary part of the instrument and the moving part which is between the two permanent magnets that consists of pointer. The deflection of the moving coil is directly proportion to the current. Thus the torque is proportional to the current which is given by the expression $T_d = K.I$, where T_d is the deflecting torque. K is proportionality constant which depends upon the strength and the number of turns in the coil. The pointer deflects between the two opposite forces produced by the spring and the magnets. And the resulting direction of the pointer is in the direction of the resultant force. The value of current is measured by the deflection angle θ, and the value of K.

Deflecting | Controlling | Damping Torque

In order to ensure proper operation of indicating instruments, the following three torques are required:

- Deflecting (or operating) torque.
- Controlling (or restoring) torque.
- Damping torque.

Deflecting Torque

One important requirement in indicating instruments is the arrangement for producing operating or deflecting torque (T_d) when the instrument is connected in the circuit to measure the given electrical quantity. This is achieved by utilizing the various effects of electric current or voltage. The deflecting torque causes the moving system to move from its zero position. The deflecting torque is produced by utilizing one or more of the following effects of current or voltage:

1. Magnetic effect ------------------------------- Moving-iron instruments.
2. Electrodynamic effect ---------------------- (i) Moving coil instruments, (ii) Dynamometer type.
3. Electromagnetic induction effect ---------Induction type instruments.
4. Thermal effect ---------------------------------Hot-wire instruments.
5. Chemical effect --------------------------------Electrolytic instruments.
6. Electrostatic effect ----------------------------Electrostatic voltmeters

The table below gives information about the electrical measuring instruments in which deflecting torque is produced by utilizing the first three effects.

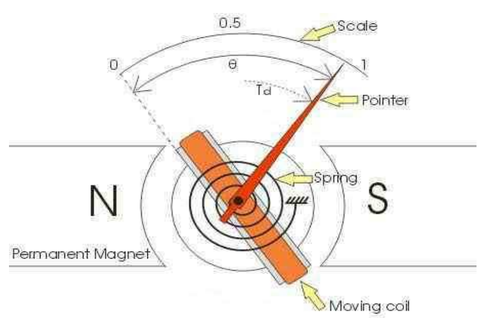
Controlling Torque

The controlling torque (T_c) opposes the deflecting torque and increases with the deflection of the moving system. The pointer comes to rest at a position where the two opposing torques are equal i.e. $T_d = T_c$. The controlling torque performs two functions.

- Controlling torque increases with the deflection of the moving system so that the final position of the pointer on the scale will be according to the magnitude of an electrical quantity (i.e. current or voltage or power) to be measured.
- Controlling torque brings the pointer back to zero when the deflecting torque is removed. If it were not provided, the pointer once deflected would not return to zero position on removing the deflecting torque. The *controlling torque* in indicating instruments may be provided by one of the following two methods:

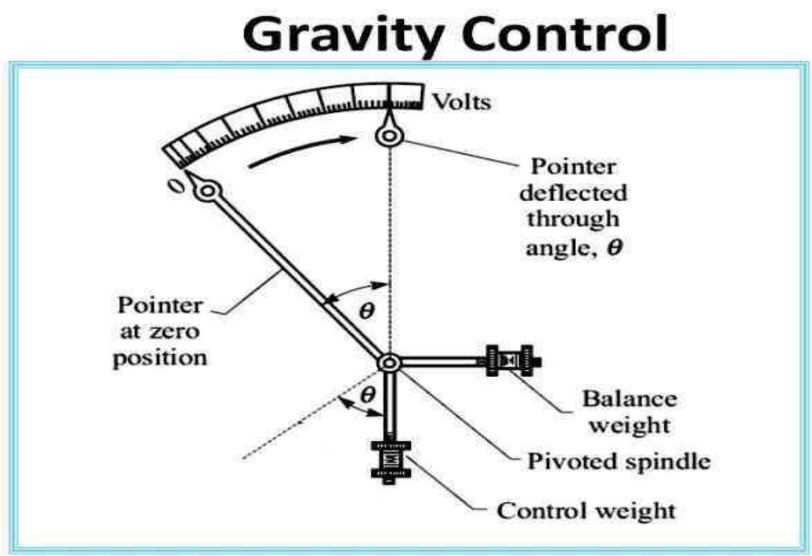
(i) Spring control.
(ii) Gravity control.

Spring Control Method



This is the most common method of providing controlling torque, in electrical instruments. A spiral hairspring made of some non-magnetic material like phosphor bronze is attached to the moving system of the instrument as shown in the figure.Springs also serve the additional purpose of leading current to the moving system (i.e. operating coil). With that deflection of the pointer, the spring is twisted in the opposite direction. This twist in the spring provides the controlling torque.Since the torsion torque of a spiral spring is proportional to the angle of twist, the controlling torque (T_c) is directly proportional to the angle of deflection of pointer (θ) i.e. $T_c \alpha \theta$. The pointer will come to rest at a position where controlling torque is equal to the deflecting torque i.e. $T_d = T_c$. In an instrument where the deflecting torque is uniform, spring control provides a uniform scale over the whole range. The balance weight is attached to counterbalance the weight of the pointer and other moving parts.

Gravity Control Method



In this method, a small weight is attached to the moving system, which provides necessary controlling torque. In the zero position of the pointer, the control weight hangs vertically downward and therefore provides no controlling torque. However, under the action of deflecting torque, the pointer moves from zero position and control weight moves in opposite direction. Due to gravity, the control weight would tend to come in original position (i.e. vertical) and thus provides an opposing or controlling torque. The pointer comes to rest at a position where controlling torque is equal to the deflecting torque. In this method, controlling torque (T_c) is proportional to the sin of angle of deflection (θ) i.e. $T_c \propto \sin\theta$.
Because in this method controlling torque (T_c) is not directly proportional to the angle of deflection (θ) but it is proportional to sin θ therefore, gravity control instruments have non-uniform scales; being crowded in beginning.

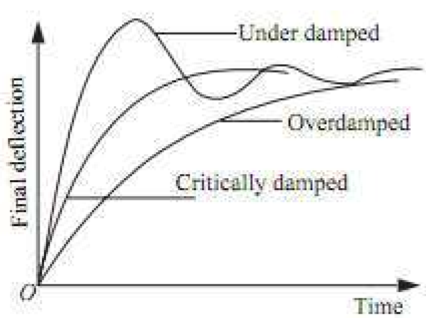
Damping Torque

A damping torque *i*s produced by a damping or stopping force which acts on the moving system only when it is moving and always opposes its motion. Such a torque is necessary to bring the pointer to rest quickly. If there is no damping torque, then the pointer will keep moving to and fro about its final deflected position for some time before coming to rest, due to the inertia of the moving system.

This damping torque acts only when the pointer is in motion and always opposes the motion. The position of the pointer when stationary is, therefore, not affected by damping torque. The degree of damping decides the behavior of the moving system.

If the instrument is under-damped, the pointer will oscillate about the final position for some time before coming to rest. On the other hand, if the instrument is over damped, the pointer will become slow and lethargic.

However, if the degree of damping is adjusted to such a value that the pointer comes up to the correct reading quickly without oscillating about it, the instrument is said to be critically damped. The damping torque in indicating instruments can be

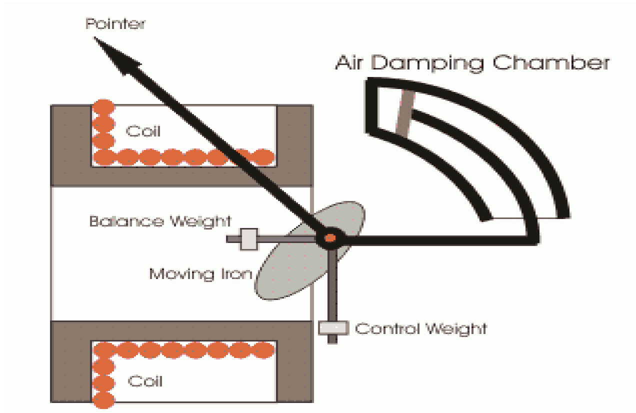


Dynamic response of a measuring instrument

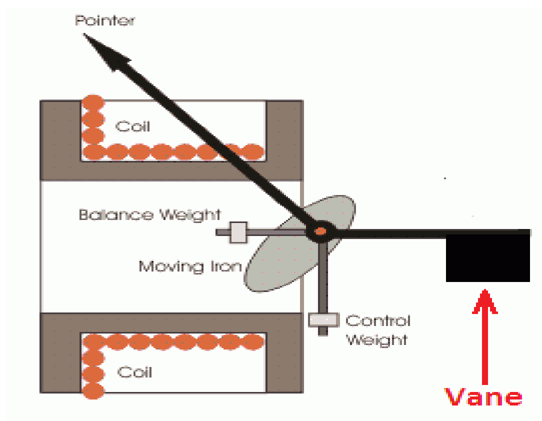
- Air friction damping.
- Fluid friction damping.
- Eddy current damping.

Air Friction Damping

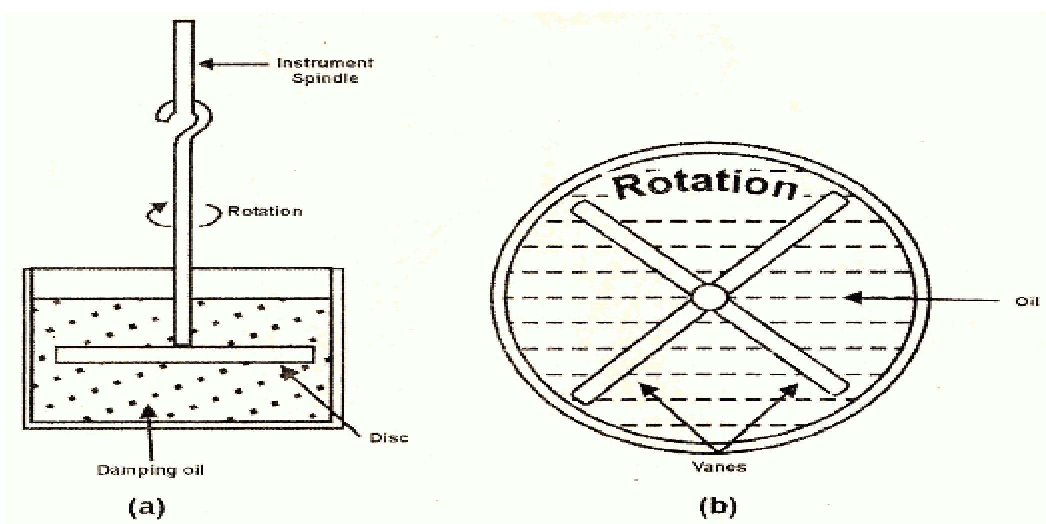
Arrangements of air friction damping are shown in fig. (a) and fig. (b). In the arrangement shown in fig (a), a light aluminum piston is attached to the spindle that carries the pointer and moves with a very little clearance in a rectangular or circular air chamber closed at one end. The cushioning action of the air on the piston damps out any tendency of the pointer to oscillate about the final deflected position. This method is not favored these days and the one shown in fig. (b) is preferred.



In this method, one or two light aluminum vanes are attached to the same spindle that carries the pointer. As the pointer moves, the vanes swing and compress the air. The pressure of compressed air on the vanes provides the necessary damping force to reduce the tendency of the pointer to oscillate.

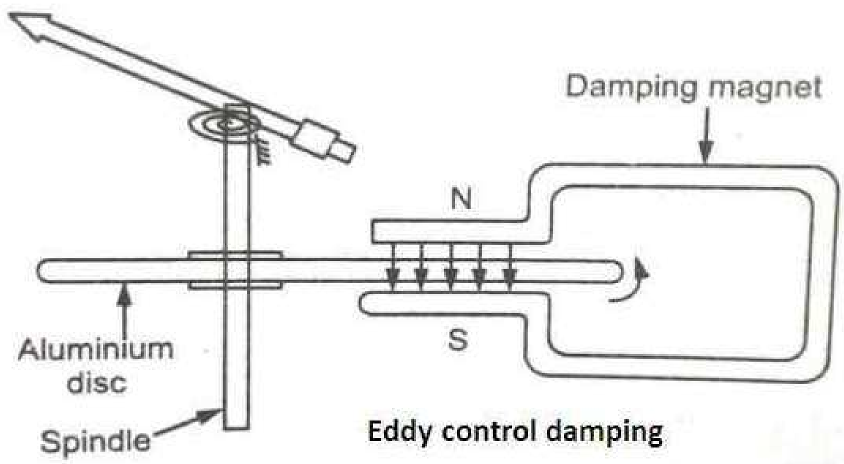


Fluid Friction Damping



In this method, discs or vanes attached to the spindle of the moving system are kept immersed in a pot containing oil of high viscosity. As the pointer moves, the friction between the oil and vanes opposes the motion of the pointer and thus necessary damping is provided. The fluid friction damping method is not suitable for portable instruments because of the oil contained in the instrument. In general, fluid friction damping is not employed in indicating instrument, although one can find its use in Kelvin electrostatic voltmeter.

Eddy Current Damping



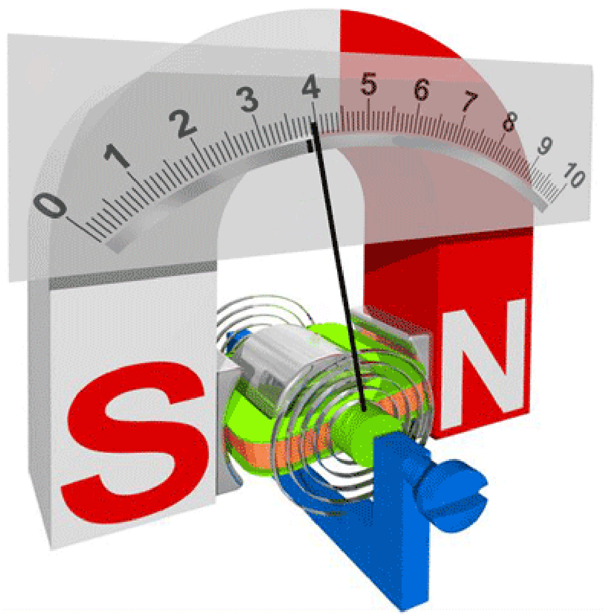
Eddy control damping

Two methods of eddy current damping are generally used.In the first method, as shown in the figure, a thin aluminum or copper disc is attached to the moving system is allowed to pass between the poles of a permanent magnet. As the pointer moves, the disc cuts across the magnetic field and eddy currents are induced in the disc. These eddy currents react with the field of the magnet to produce a force which opposes the motion according to Lenz's Law. In this way, eddy current damping torque reduces the oscillations of the pointer. In the second method, the coil which produces the deflecting torque is wound on aluminum former. As coil moves in the field of the instrument, eddy currents are induced in the aluminum former to provide the necessary damping torque.

Permanent Magnet Moving Coil Instrument

The permanent magnet moving coil instrument or PMMC type instrument uses two permanent magnets in order to create stationary magnetic field. These types of instruments are only used for measuring the DC quantities as if we apply AC current to these type of instruments the direction of current will be reversed during negative half cycle and hence the direction of torque will also be reversed which gives average value of torque zero. The pointer will not deflect due to high frequency from its mean position showing zero reading. However it can measure the direct current very accurately.

Let us move towards the constructions of permanent magnet moving coil instruments. We will see the construction of these types of instruments in five parts and they are described below:



- **Stationary Part or Magnet System:**

  In the present time we use magnets of high field intensities, high coercive force instead of using U shaped permanent magnet having soft iron pole pieces. The magnets which we are using nowadays are made up of materials like alcomax and alnico which provide high field strength.

- **Moving Coil:**

  The moving coil can freely moves between the two permanent magnets as shown in the figure given below. The coil is wound with many turns of copper wire and is placed on rectangular aluminum which is pivoted on jeweled bearings.

- **Control System:**

  The spring generally acts as control system for PMMC instruments. The spring also serves another important function by providing the path to lead current in and out of the coil.

- **Damping System:**

  The damping force hence torque is provided by movement of aluminum former in the magnetic field created by the permanent magnets.

- **Meter:**

  Meter of these instruments consists of light weight pointer to have free movement and scale which is linear or uniform and varies with angle.

Let us derive a general expression for torque in permanent magnet moving coil instruments or PMMC instruments. We know that in moving coil instruments the deflecting torque is given by the expression:

Td = NBldI where N is number of turns,

B is magnetic flux density in air gap,

l is the length of moving coil,

d is the width of the moving coil,

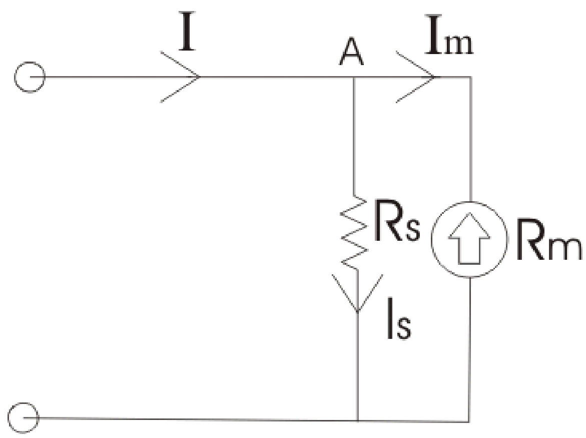
And I is the electric current.

Now for a moving coil instruments deflecting torque should be proportional to current, mathematically we can write Td = GI. Thus on comparing we say G = NBIdl. At steady state we have both the controlling and deflecting torques are equal. Tc is controlling torque, on equating controlling torque with deflection torque we have GI = K.x where x is deflection thus current is given

$$I = \frac{K}{G}x$$

Since the deflection is directly proportional to the current therefore we need a uniform scale on the meter for measurement of current.

Now we are going to discuss about the basic circuit diagram of the ammeter. Let us consider a circuit as shown below:



The current I is shown which breaks into two components at the point A. The two components are I_s and I_m. Before I comment on the magnitude values of these currents, let us know more about the construction of shunt resistance. The basic properties of shunt resistance are written below,

The electrical resistance of these shunts should not differ at higher temperature, it they should posses very low value of temperature coefficient. Also the resistance should be time independent. Last and the most important property they should posses is that they should be able to carry high value of current without much rise in temperature. Usually manganin is used for making DC resistance. Thus we can say that the value of I_s much greater than the value of I_m as resistance of shunt is low. From the we have,

$$I_s \cdot R_s = I_m \cdot R_m$$

Where, R_s is resistance of shunt and R_m is the electrical resistance of the coil.

$$Also\ I_s = I - I_m$$

From the above two equations we can write,

$$m = \frac{I}{I_m} = 1 + \frac{R_m}{R_s}$$

Where, m is the magnifying power of the shunt.

Errors in Permanent Magnet Moving Coil Instruments

There are three main types of errors:

1. Errors due to permanent magnets: Due to temperature effects and aging of the magnets the magnet may lose their magnetism to some extent. The magnets are generally aged by the heat and vibration treatment.
2. Error may appear in PMMC Instrument due to the aging of the spring. However the error caused by the aging of the spring and the errors caused due to permanent magnet are opposite to each other, hence both the errors are compensated with each other.
3. Change in the resistance of the moving coil with the temperature: Generally the temperature coefficients of the value of coefficient of copper wire in moving coil is 0.04 per degree Celsius rise in temperature. Due to lower value of temperature coefficient the temperature rises at faster rate and hence the resistance increases. Due to this significant amount of error is caused.

Advantages of Permanent Magnet Moving Coil Instruments

1. The scale is uniformly divided as the current is directly proportional to deflection of the pointer. Hence it is very easy to measure quantities from these instruments.
2. Power consumption is also very low in these types of instruments.
3. Higher value of torque is to weight ratio.
4. These are having multiple advantages, a single instrument can be used for measuring various quantities by using different values of shunts and multipliers.

Instead of various advantages the permanent magnet moving coil instruments or PMMC Instrument posses few disadvantages.

Disadvantages of Permanent Magnet Moving Coil Instruments

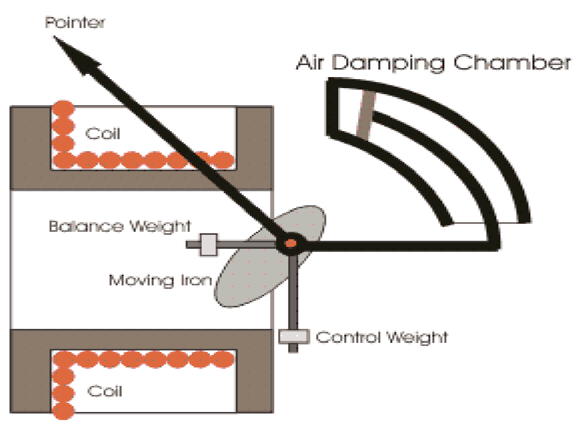
1. These instruments cannot measure ac quantities.
2. Cost of these instruments is high as compared to moving iron instruments.

Moving Iron Instrument

This instrument is one of the most primitive forms of measuring and relay instrument. Moving iron type instruments are of mainly two types. Attraction type and repulsion type instrument. Whenever a piece of iron is placed nearer to a magnet it would be attracted by the magnet. The force of this attraction depends upon the strength said magnetic field. If the magnet is electromagnet then the magnetic field strength can easily be increased or decreased by increasing or decreasing current through its coil. Accordingly the attraction force acting on the piece of iron would also be increased and decreased. Depending upon this simple phenomenon attraction type moving iron instrument was developed.

Whenever two pieces of iron are kept side by side and a magnet is brought nearer to them the iron pieces will repulse each other. This repulsion force is due to same magnetic poles induced in same sides the iron pieces due external magnetic field. This repulsion force increases if field strength of the magnet is increased. Like case if the magnet is electromagnet, then magnetic field strength can easily be controlled by controlling input current to the magnet. Hence if the current increases the repulsion force between the pieces of iron is increased and it the current decreases the repulsion force between them is decreased. Depending upon this phenomenon repulsion type moving iron instrument was constructed.

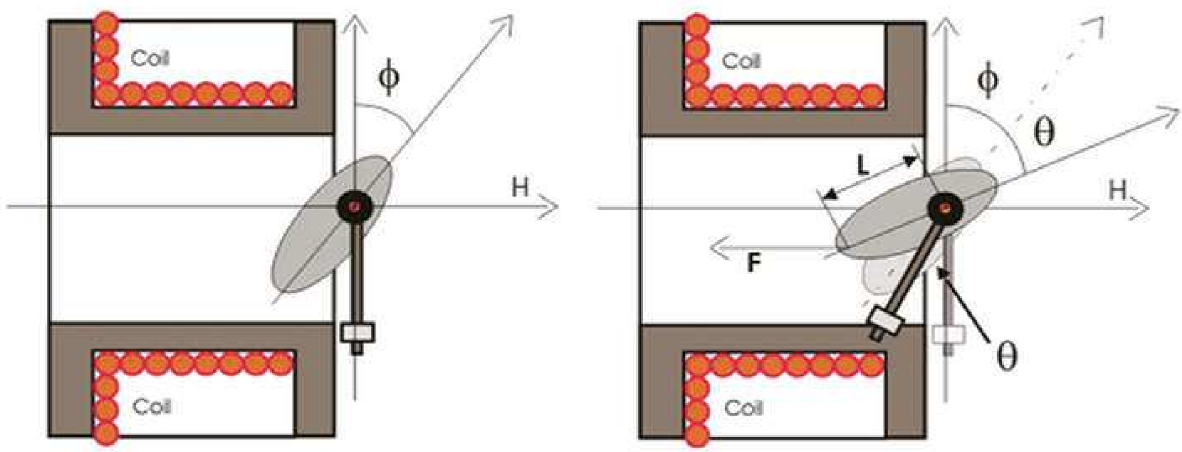
Construction of Moving Iron Instrument



The basic construction of attraction type moving iron instrument is illustrated bellow A thin disc of soft iron is eccentrically pivoted in front of a coil. This iron tends to move inward that is from weaker magnetic field to stronger magnetic field when current flowing through the coil. In attraction moving instrument gravity control was used previously but now gravity control method is replaced by spring control in relatively modern instrument. By adjusting balance weight null deflection of the pointer is achieved. The required damping force is provided in this instrument by air friction. The figure shows a typical type of damping system provided in the instrument, where damping is achieved by a moving piston in an air syringe.

Theory of Attraction Type Moving Iron Instrument

Suppose when there is no current through the coil, the pointer is at zero, the angle made by the axis of the iron disc with the line perpendicular to the field is φ. Now due current I and corresponding magnetic field strength, the iron piece is deflected to an angle θ. Now component of H in the direction of defected iron disc axis is Hcos{90 - (θ + φ) or Hsin (θ + φ). Now force F acting on the disc inward to the coil is thus proportional to $H^2\sin(\theta + \varphi)$ hence the force is also proportional to $I^2\sin(\theta + \varphi)$ for constant permeability. If this force is acting on the disc at a distance l from the pivot, then deflection torque,



$$T_d = Fl\cos(\theta + \phi)$$

$$Thus\, T_d \propto I^2\sin(\theta + \phi)\cos(\theta + \phi) \propto I^2\sin2(\theta + \phi)$$

Since I is constant.

$$T_d = kI^2 sin2(\theta + \phi)$$

Where, k is constant.

Now, as the instrument is gravity controlled, controlling torque will be

$$T_c = k' sin\theta$$

Where, k' is constant.

At steady state condition,

$$T_d = T_c \Rightarrow kI^2 sin2(\theta + \phi) = k' sin\theta$$

$$\Rightarrow I = \sqrt{\frac{k' sin\theta}{k sin2(\theta + \phi)}} = K\sqrt{\frac{sin\theta}{sin2(\theta + \phi)}}$$

Where, K is constant.

UNIT-II

DC MACHINES

Principle of DC Generator

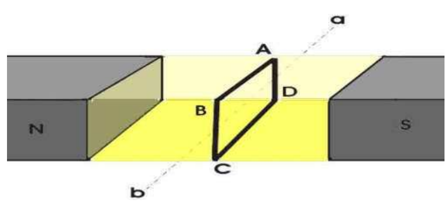
There are two types of generators, one is ac generator and other is DC generator. Whatever may be the types of generators, it always converts mechanical power to electrical power. An AC generator produces alternating power. A DC generator produces direct power. Both of these generators produce electrical power, based on same fundamental principle of Faraday's law of electromagnetic induction. According to this law, when a conductor moves in a magnetic field it cuts magnetic lines of force, due to which an emf is induced in the conductor. The magnitude of this induced emf depends upon the rate of change of flux (magnetic line force) linkage with the conductor. This emf will cause a current to flow if the conductor circuit is closed.

Hence the most basic tow essential parts of a generator are

1. a magnetic field
2. conductors which move inside that magnetic field.

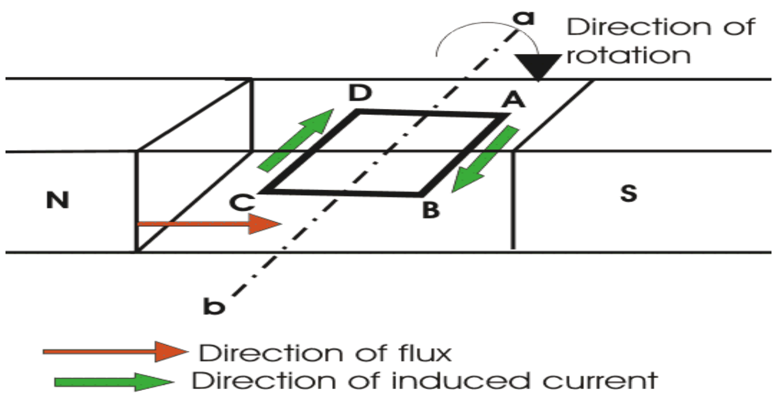
Now we will go through working principle of DC generator. As, the working principle of ac generator is not in scope of our discussion in this section.

Single Loop DC Generator

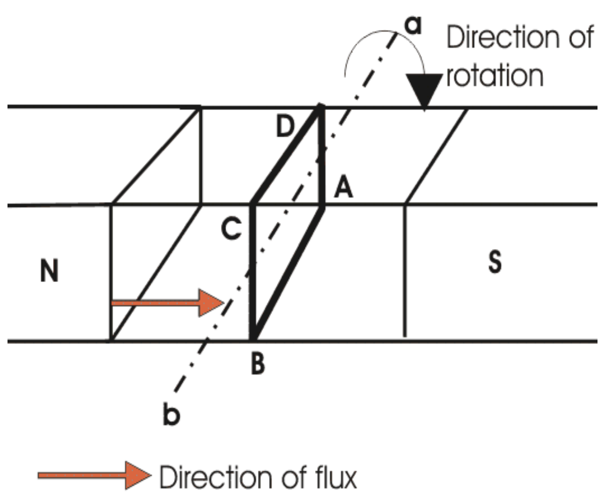


In the figure above, a single loop of conductor of rectangular shape is placed between two opposite poles of magnet.

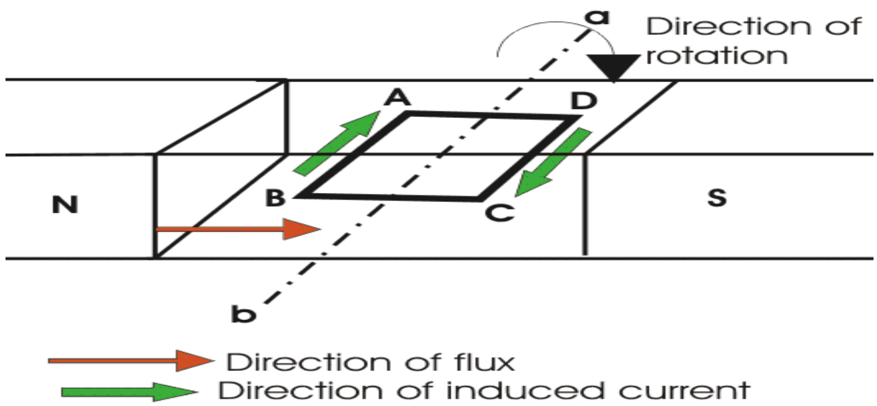
Let's us consider, the rectangular loop of conductor is ABCD which rotates inside the magnetic field about its own axis ab. When the loop rotates from its vertical position to its horizontal position, it cuts the flux lines of the field. As during this movement two sides, i.e. AB and CD of the loop cut the flux lines there will be an emf induced in these both of the sides (AB and BC) of the loop.



As the loop is closed there will be a current circulating through the loop. The direction of the current can be determined by Flemming's right hand Rule. This rule says that if you stretch thumb, index finger and middle finger of your right hand perpendicular to each other, then thumbs indicates the direction of motion of the conductor, index finger indicates the direction of magnetic field i.e. N - pole to S - pole, and middle finger indicates the direction of flow of current through the conductor. Now if we apply this right hand rule, we will see at this horizontal position of the loop, current will flow from point A to B and on the other side of the loop current will flow from point C to D.



Now if we allow the loop to move further, it will come again to its vertical position, but now upper side of the loop will be CD and lower side will be AB (just opposite of the previous vertical position). At this position the tangential motion of the sides of the loop is parallel to the flux lines of the field. Hence there will be no question of flux cutting and consequently there will be no current in the loop.
If the loop rotates further, it comes to again in horizontal position. But now, said AB side of the loop comes in front of N pole and CD comes in front of S pole, i.e. just opposite to the previous horizontal position as shown in the figure beside.

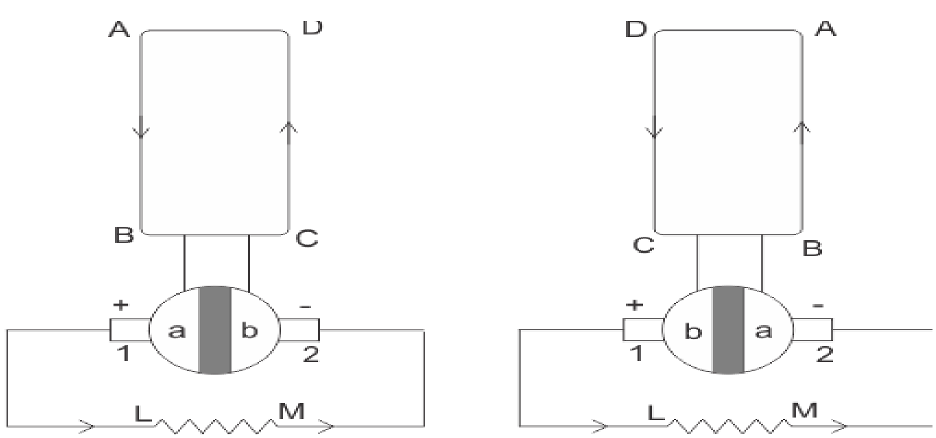


Here the tangential motion of the side of the loop is perpendicular to the flux lines, hence rate of flux cutting is maximum here and according to Fleming's right hand rule, at this position current flows from B to A and on other side from D to C.Now if the loop is continued to rotate about its axis, every time the side AB comes in front of S pole, the current flows from A to B and when it comes in front of N pole, the current flows from B to A. Similarly, every time the side CD comes in front of S pole the current flows from C to D and when it comes in front of N pole the current flows from D to C.

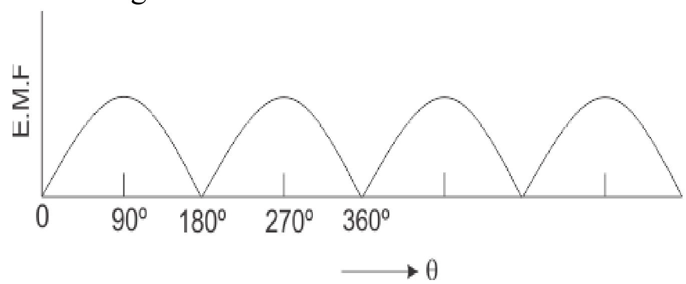
If we observe this phenomena in different way, it can be concluded, that each side of the loop comes in front of N pole, the current will flow through that side in same direction i.e. downward to the reference plane and similarly each side of the loop comes in front of S pole, current through it flows in same direction i.e. upwards from reference plane. From this, we will come to the topic of principle of DC generator.

Now the loop is opened and connected it with a split ring as shown in the figure below. Split ring are made out of a conducting cylinder which cuts into two halves or segments insulated from each other. The external load terminals are connected with two carbon brushes which are rest on these split slip ring segments.

Working Principle of DC Generator



It is seen that in the first half of the revolution current flows always along ABLMCD i.e. brush no 1 in contact with segment a. In the next half revolution, in the figure the direction of the induced current in the coil is reversed. But at the same time the position of the segments a and b are also reversed which results that brush no 1 comes in touch with the segment b. Hence, the current in the load resistance again flows from L to M. The wave from of the current through the load circuit is as shown in the figure. This current is unidirectional.



This is basic working principle of DC generator, explained by single loop generator model. The position of the brushes of DC generator is so arranged that the change over of the segments a and b from one brush to other takes place when the plane of rotating coil is at right angle to the plane of the lines of force. It is so become in that position, the induced emf in the coil is zero.

Construction of DC Generator

During explaining working principle of DC Generator, we have used a single loop DC generator. But now we will discuss about practical construction of DC Generator. A DC generator has the following parts

1. Yoke
2. Pole of generator
3. Field winding
4. Armature of DC generator
5. Brushes of generator and Commentator
6. Bearing

Yoke of DC Generator

Yoke or the outer frame of DC generator serves two purposes,

1. It holds the magnetic pole cores of the generator and acts as cover of the generator.
2. It carries the magnetic field flux.

In small generator, yoke are made of cast iron. Cast iron is cheaper in cost but heavier than steel. But for large construction of DC generator, where weight of the machine is concerned, lighter cast steel or rolled steel is preferable for constructing yoke of DC generator. Normally larger yokes are formed by rounding a rectangular steel slab and the edges are welded together at the bottom. Then feet, terminal box and hangers are welded to the outer periphery of the yoke frame.

Pole Cores and Pole Shoes

Let's first discuss about pole core of DC generator. There are mainly two types of construction available.

One: Solid pole core, where it is made of a solid single piece of cast iron or cast steel.

Two: Laminated pole core, where it made of numbers of thin, limitations of annealed steel which are riveted together.

The thickness of the lamination is in the range of 0.04" to 0.01". The pole core is fixed to the inner periphery of the yoke by means of bolts through the yoke and into the pole body. Since the poles project inwards they are called salient poles.

The pole shoes are so typically shaped, that, they spread out the magnetic flux in the air gap and reduce the reluctance of the magnetic path.

Due to their larger cross-section they hold the pole coil at its position.

Pole Coils: The field coils or pole coils are wound around the pole core. These are a simple coil of insulated copper wire or strip, which placed on the pole which placed between yoke and pole shoes as shown.

Armature Core

The purpose of armature core is to hold the armature winding and provide low reluctance path for the flux through the armature from N pole to S pole. Although a DC generator provides direct current but induced current in the armature is alternating in nature. That is why, cylindrical or drum shaped armature core is build up of circular laminated sheet. In every circular lamination, slots are either die - cut or punched on the outer periphery and the key way is located on the inner periphery as shown. Air ducts are also punched of cut on each lamination for circulation of air through the core for providing better cooling. Up to diameter of 40", the circular stampings are cut out in one piece of lamination sheet. But above 40", diameter, number of suitable sections of a circle is cut. A complete circle of lamination is formed by four or six or even eight such segment.

Armature Winding

Armature winding are generally formed wound. These are first wound in the form of flat rectangular coils and are then pulled into their proper shape in a coil puller. Various conductors of the coils are insulated from each other. The conductors are placed in the armature slots, which are lined with tough insulating material. This slot insulation is folded over above the armature conductors placed in it and secured in place by special hard wooden or fiber wedges. Two types of armature windings are used – Lap winding and Wave winding.

Commutator

The commentator plays a vital role in DC generator. It collects current from armature and sends it to the load as direct current. It actually takes alternating current from armature and converts it to direct current and then send it to external load. It is cylindrical structured and is build up of wedge-shaped segments of high conductivity, hard drawn or drop forged copper. Each segment is insulated from the shaft by means of insulated commutator segment shown below. Each commentator segment is connected with corresponding armature conductor through segment riser or lug.

Brushes

The brushes are made of carbon. These are rectangular block shaped. The only function of these carbon brushes of DC generator is to collect current from commutator segments. The brushes are housed in the rectangular box shaped brush holder or brush box. As shown in figure, the brush face is placed on the commutator segment which is attached to the brush holder.

Bearing

For small machine, ball bearing is used and for heavy duty DC generator, roller bearing is used. The bearing must always be lubricated properly for smooth operation and long life of generator.

Armature winding

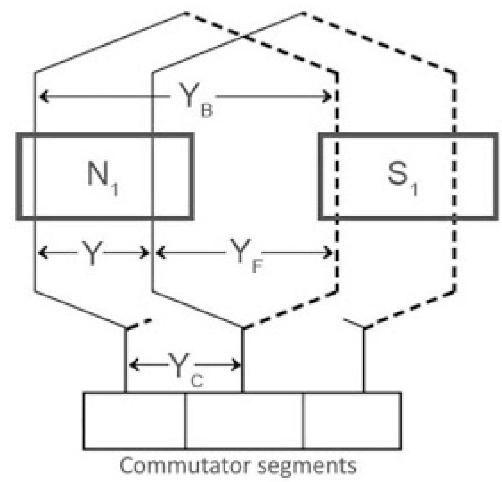
Basically armature winding of a DC machine is wound by one of the two methods, lap winding or wave winding. The difference between these two is merely due to the end connections and commutator connections of the conductor. To know how armature winding is done, it is essential to know the following terminologies -

1. Pole pitch: It is defined as number of armature slots per pole. For example, if there are 36 conductors and 4 poles, then the pole pitch is 36/4=9.
2. Coil span or coil pitch (Ys): It is the distance between the two sides of a coil measured in terms of armature slots.
3. Front pitch (Yf): It is the distance, in terms of armature conductors, between the second conductor of one coil and the first conductor of the next coil. OR it is the distance between two coil sides that are connected to the same commutator segment.
4. Back pitch (Yb): The distance by which a coil advances on the back of the armature is called as back pitch of the coil. It is measured in terms of armature conductors.
5. Resultant pitch (Yr): The distance, in terms of armature conductor, between the beginning of one coil and the beginning of the next coil is called as resultant pitch of the coil.

Armature winding can be done as single layer or double layer. It may be simplex, duplex or multiplex, and this multiplicity increases the number of parallel paths.

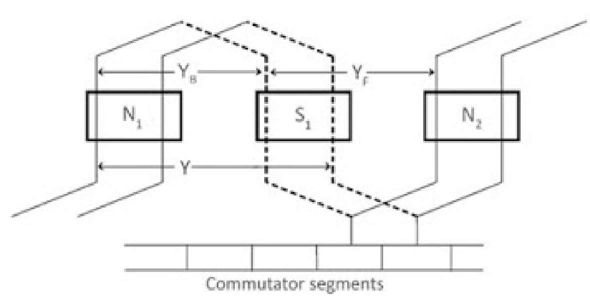
Lap Winding and Wave Winding

In lap winding, the successive coils overlap each other. In a simplex lap winding, the two ends of a coil are connected to adjacent commutator segments. The winding may be progressive or retrogressive. A progressive winding progresses in the direction in which the coil is wound. The opposite way is retrogressive. The following image shows progressive simplex lap winding.



Lap Winding

In wave winding, a conductor under one pole is connected at the back to a conductor which occupies an almost corresponding position under the next pole which is of opposite polarity. In other words, all the coils which carry e.m.f in the same direction are connected in series. The following diagram shows a part of simplex wave winding.



Wave winding

Basis For Comparison	Lap Winding	Wave Winding
Definition	The coil is lap back to the	The coil of the winding form

	succeeding coil.	the wave shape.
Connection	The end of the armature coil is connected to an adjacent segment on the commutators.	The end of the armature coil is connected to commutator segments some distance apart.
Parallel Path	The numbers of parallel path are equal to the total of number poles.	The number of parallel paths is equal to two.
Other Name	Parallel Winding or Multiple Winding	Two-circuit or Series Winding.
EMF	Less	More
Number of Brushes	Equal to the number of parallel paths.	Two
Types	Simplex and Duplex lap winding.	Progressive and Retrogressive wave winding
Efficiency	Less	High
Additional Coil	Equalizer Ring	Dummy coil
Winding Cost	High (because more conductor is required)	Low
Uses	In low voltage, high current machines.	In high voltage, low current machines.

EMF Equation of a DC Generator

Consider a DC generator with the following parameters,

P = number of field poles
Ø = flux produced per pole in Wb (weber)
Z = total no. of armature conductors
A = no. of parallel paths in armature
N = rotational speed of armature in revolutions per min. (rpm)

Now,

- Average emf generated per conductor is given by dΦ/dt (Volts) ... eq. 1
- Flux cut by one conductor in one revolution = dΦ = PΦ ….(Weber),
- Number of revolutions per second (speed in RPS) = N/60
- Therefore, time for one revolution = dt = 60/N (Seconds)
- From eq. 1, emf generated per conductor = dΦ/dt = PΦN/60 (Volts) …..(eq. 2)

Above equation-2 gives the emf generated in one conductor of the generator. The conductors are connected in series per parallel path, and the emf across the generator terminals is equal to the generated emf across any parallel path.

Therefore, Eg = PΦNZ / 60A

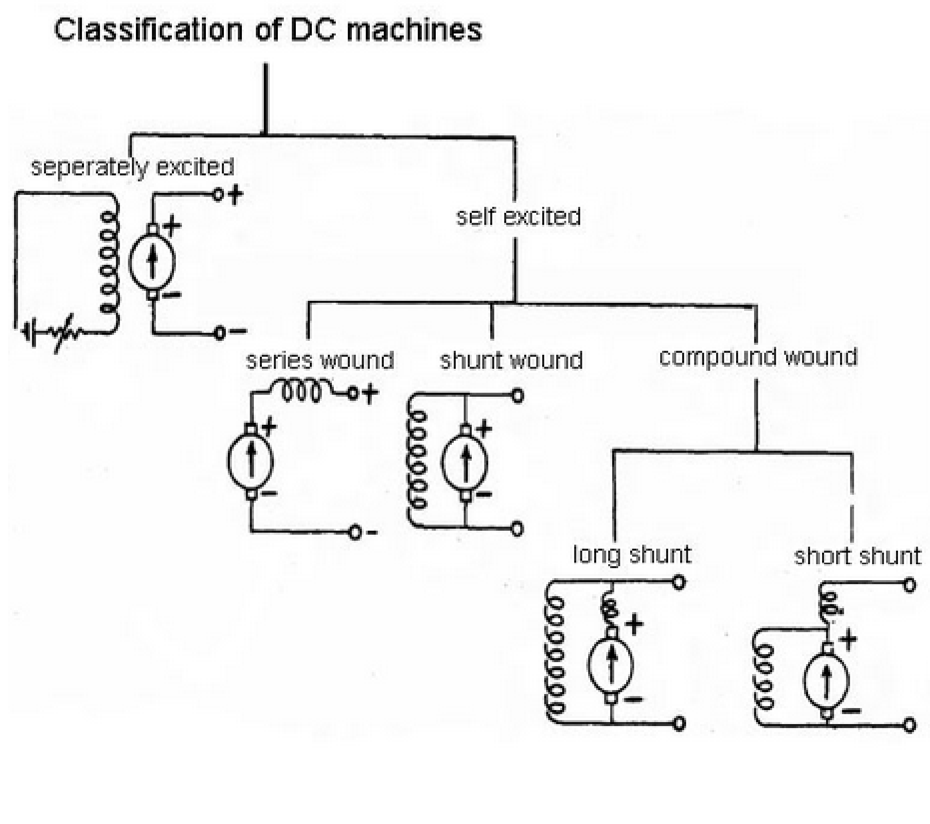
For simplex lap winding, number of parallel paths is equal to the number of poles (i.e. A=P), Therefore, for simplex lap wound dc generator, Eg = PΦNZ / 60P

For simplex wave winding, number of parallel paths is equal to 2 (i.e P=2), Therefore, for simplex wave wound dc generator, Eg = PΦNZ / 120

Types of DC Generators

Generally DC generators are classified according to the ways of excitation of their fields. There are three methods of excitation.

1. Field coils excited by permanent magnets – Permanent magnet DC generators.
2. Field coils excited by some external source – Separately excited DC generators.
3. Field coils excited by the generator itself – Self excited DC generators.



A brief description of these type of generators are given below.

Separately Excited DC Generator

These are the generators whose field magnets are energized by some external DC source such as battery.

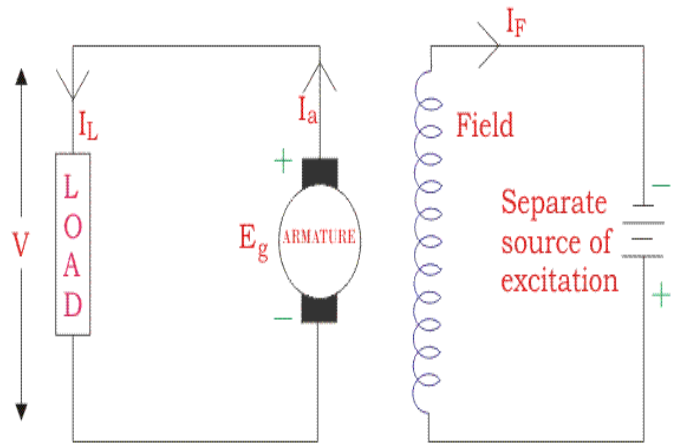
A circuit diagram of separately excited DC generator is shown in figure.

I_a = Armature current

I_L = Load current

V = Terminal voltage

E_g = Generated emf



Separately Excited DC Generator

Voltage drop the armature = $I_a \times R_a$

Let,

$$I_a = I_L = I\ (say)$$

Then,

$$voltage\ across\ the\ load, V = IR_a$$

Power generated,

$$P_g = E_g \times I$$

Power delivered to the external load,

$$P_L = V \times I$$

Self-excited DC Generators

These are the generators whose field magnets are energized by the current supplied by themselves. In these type of machines field coils are internally connected with the armature. Due to residual magnetism some flux is always present in the poles. When the armature is rotated some emf is induced. Hence some induced current is produced. This small current flows through the field coil as well as the load and thereby strengthening the pole flux. As the pole flux strengthened, it will produce more armature emf, which cause further increase of current through the field. This increased field current further raises armature emf and this cumulative phenomenon continues until the excitation reaches to the rated value. According to the position of the field coils the self-excited DC generators may be classified as…

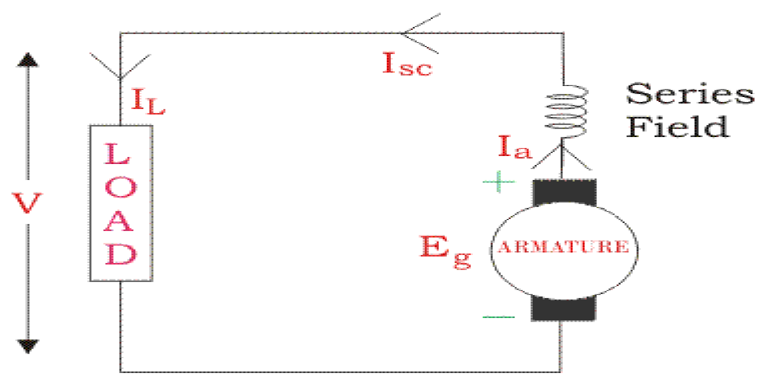
1. Series wound generators
2. Shunt wound generators
3. Compound wound generators

Series Wound Generator

In these types of generators, the field windings are connected in series with armature conductors as shown in figure below. So, whole current flows through the field coils as well as the load. As series field winding carries full load current it is designed with relatively few turns of thick wire. The electrical resistance of series field winding is therefore very low (nearly 0.5Ω).

Let,

R_{sc} = Series winding resistance
I_{sc} = Current flowing through the series field
R_a = Armature resistance
I_a = Armature current
I_L = Load current
V = Terminal voltage
E_g = Generated emf



Series Wound Generator

Then,

$$I_a = I_{sc} = I_L = I\ (say)$$

Voltage across the load,

$$V = E_g - I(I_a \times R_a)$$

Power generated,

$$P_g = E_g \times I$$

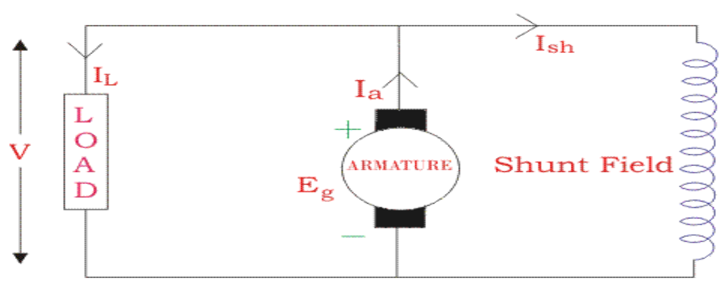
Power delivered to the load,

$$P_L = V \times I$$

Shunt Wound DC Generators

In these type of DC generators the field windings are connected in parallel with armature conductors as shown in figure below. In shunt wound generators the voltage in the field winding is same as the voltage across the terminal. Let, R_{sh} = Shunt winding resistance.

I_{sh} = Current flowing through the shunt field
R_a = Armature resistance
I_a = Armature current
I_L = Load current
V = Terminal voltage
E_g = Generated emf



Shunt Wound Generator

Here armature current I_a is dividing in two parts, one is shunt field current I_{sh} and another is load current I_L.

$$I_a = I_{sh} + I_L$$

The effective power across the load will be maximum when I_L will be maximum. So, it is required to keep shunt field current as small as possible. For this purpose the resistance of the shunt field winding generally kept high (100 Ω) and large no of turns are used for the desired emf.

shunt field current

$$I_{sh} = \frac{V}{R_{sh}}$$

Voltage across the load,

$$V = E_g - I_a R_a$$

Power generated,

$$P_g = E_g \times I_a$$

Power delivered to the load,

$$P_L = V \times I_L$$

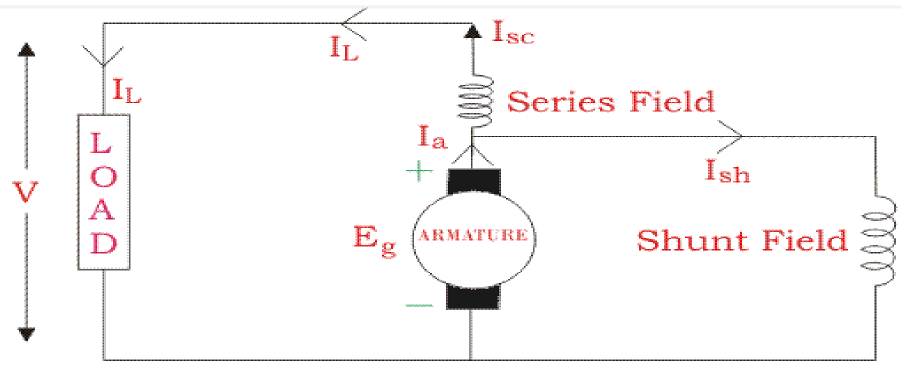
Compound Wound DC Generator

`In series wound generators, the output voltage is directly proportional with load current. In shunt wound generators, output voltage is inversely proportional with load current. A combination of these two types of generators can overcome the disadvantages of both. This combination of windings is called compound wound DC generator.

Compound wound generators have both series field winding and shunt field winding. One winding is placed in series with the armature and the other is placed in parallel with the armature. This type of DC generators may be of two types- short shunt compound wound generator and long shunt compound wound generator.

Short Shunt Compound Wound DC Generator

The generators in which only shunt field winding is in parallel with the armature winding as shown in figure.



Short Shunt Compound Wound Generator

$$I_{sc} = I_L$$

Shunt field current,

$$I_{sh} = \frac{(V + I_{sc}R_{sc})}{R_{sh}}$$

Armature current,

$$I_a = I_{sh} + I_L$$

Voltage across the load,

$$V = E_g - I_a R_a - I_{sc} R_{sc}$$

Power generated,

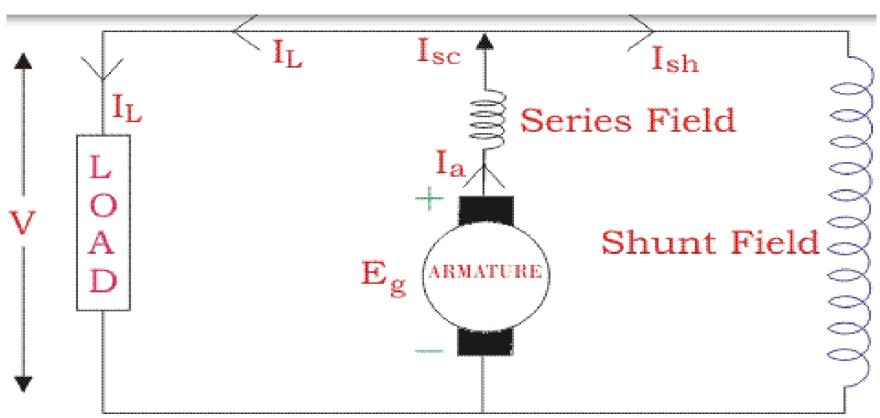
$$P_g = E_g \times I_a$$

Power delivered to the load,

$$P_L = V \times I_L$$

Long Shunt Compound Wound DC Generator

The generators in which shunt field winding is in parallel with both series field and armature winding as shown in figure.



Long Shunt Compound Wound Generator

Shunt field current,

$$I_{sh} = \frac{V}{R_{sh}}$$

Armature current, I_a = series field current,

$$I_{sc} = I_L + I_{sh}$$

Voltage across the load,

$$V = E_g - I_a R_a - I_{sc} R_{sc} = E_g - I_a(R_a + R_{sc}) \;\; [\because I_a = I_{cs}]$$

Power generated,

$$P_g = E_g \times I_a$$

Power delivered to the load,

$$P_L = V \times I_L$$

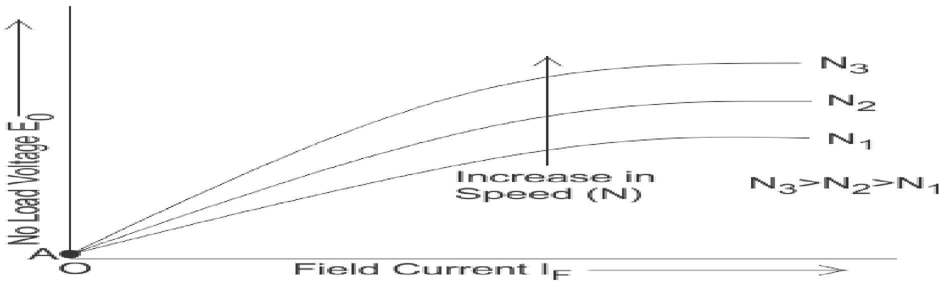
In a compound wound generator, the shunt field is stronger than the series field. When the series field assists the shunt field, generator is said to be commutatively compound wound. On the other hand if series field opposes the shunt field, the generator is said to be differentially compound wound.

Characteristic of Separately Excited DC Generator

In a separately excited DC generator, the field winding is excited by an external independent source. There are generally three most important characteristic of DC generator.

Magnetic or Open Circuit Characteristic of Separately Excited DC Generator

The curve which gives the relation between field current (I_f) and the generated voltage(E_0) in the armature on no load is called magnetic or open circuit characteristic of a DC generator. The plot of this curve is practically same for all types of generators, whether they are separately excited or self-excited. This curve is also known as no load saturation characteristic curve of DC generator. Here in this figure below we can see the variation of generated emf on no load with field current for different fixed speeds of the armature. For higher value of constant speed, the steepness of the curve is more. When the field current is zero, for the effect residual magnetism in the poles, there will be a small initial emf (OA) as show in figure.



Magnetic or Open Circuit Characteristic

Let us consider a separately excited DC generator giving its no load voltage E_0 for a constant field current. If there is no armature reaction and armature voltage drop in the machine then the voltage will remain constant. Therefore, if we plot the rated voltage on the Y axis and load current on the X axis then the curve will be a straight line and parallel to X-axis as shown in figure below. Here, AB line indicating the no load voltage (E_0). When the generator is loaded then the voltage drops due to two main reasons-

1. Due to armature reaction,
2. Due to ohmic drop (I_aR_a).

Internal or Total Characteristic of Separately Excited DC Generator

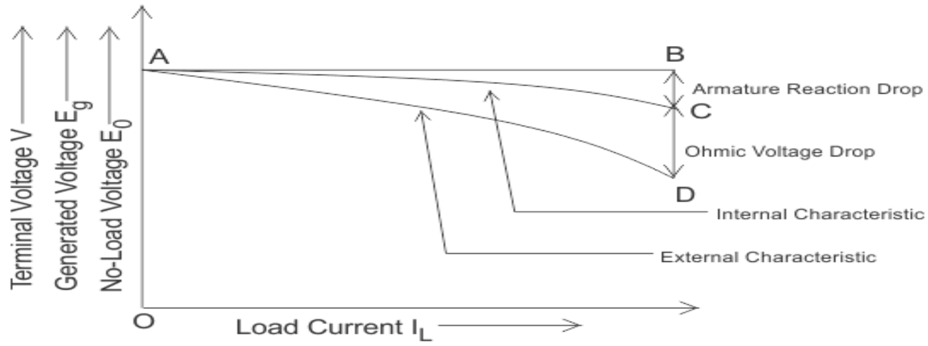
The internal characteristic of the separately excited DC generator is obtained by subtracting the drops due to armature reaction from no load voltage. This curve of actually generated voltage (E_g) will be slightly dropping. Here, AC line in the diagram indicating the actually generated voltage (E_g) with respect to load current. This curve is also called total characteristic of separately excited DC generator.

External Characteristic of Separately Excited DC Generator

The external characteristic of the separately excited DC generator is obtained by subtracting the drops due to ohmic loss ($I_a R_a$) in the armature from generated voltage (E_g).

Terminal voltage(V) = $E_g - I_a R_a$.

This curve gives the relation between the terminal voltage (V) and load current. The external characteristic curve lies below the internal characteristic curve. Here, AD line in the diagram below is indicating the change in terminal voltage(V) with increasing load current. It can be seen from figure that when load current increases then the terminal voltage decreases slightly. This decrease in terminal voltage can be maintained easily by increasing the field current and thus increasing the generated voltage. Therefore, we can get constant terminal voltage.



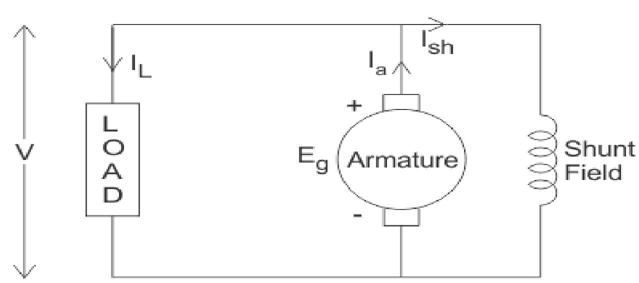
Internal and External Characteristic Curve

Separately excited DC generators have many advantages over self-excited DC generators. It can operate in stable condition with any field excitation and gives wide range of output voltage. The main disadvantage of these kinds of generators is that it is very expensive of providing a separate excitation source.

Characteristic of Shunt Wound DC Generator

In shunt wound DC generators the field windings are connected in parallel with armature conductors as shown in figure below. In these type of generators the armature current $I_a$divides in two parts. One part is the shunt field current I_{sh} flows through shunt field winding and the other part is the load current I_L goes through the external load.

$$So,\ I_a = I_{sh} + I_L$$



Shunt Wound Generator

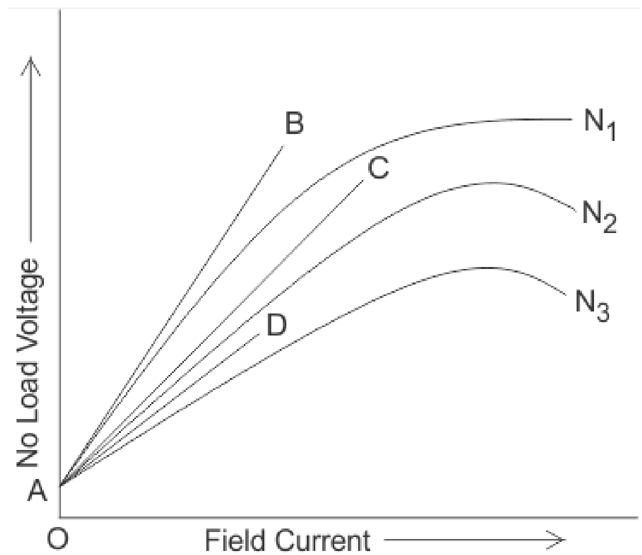
Three most important characteristic of shunt wound dc generators are discussed below:

Magnetic or Open Circuit Characteristic of Shunt Wound DC Generator

This curve is drawn between shunt field current(I_{sh}) and the no load voltage (E_0). For a given excitation current or field current, the emf generated at no load E_0 varies in proportionally with the rotational speed of the armature. Here in the diagram the magnetic characteristic curve for various speeds are drawn. Due to residual magnetism the curves start from a point A slightly up from the origin O. The upper portions of the curves are bend due to saturation. The external loadresistance of the machine needs to be maintained greater than its critical value otherwise the machine will not excite or will stop running if it is already in motion. AB, AC and AD are the slops which give critical resistances at speeds N_1, N_2 and N_3. Here, $N_1 > N_2 > N_3$.

Critical Load Resistance of Shunt Wound DC Generator

This is the minimum external load resistance which is required to excite the shunt wound generator.



Magnetic or Open Circuit Characteristic

Internal Characteristic of Shunt Wound DC Generator

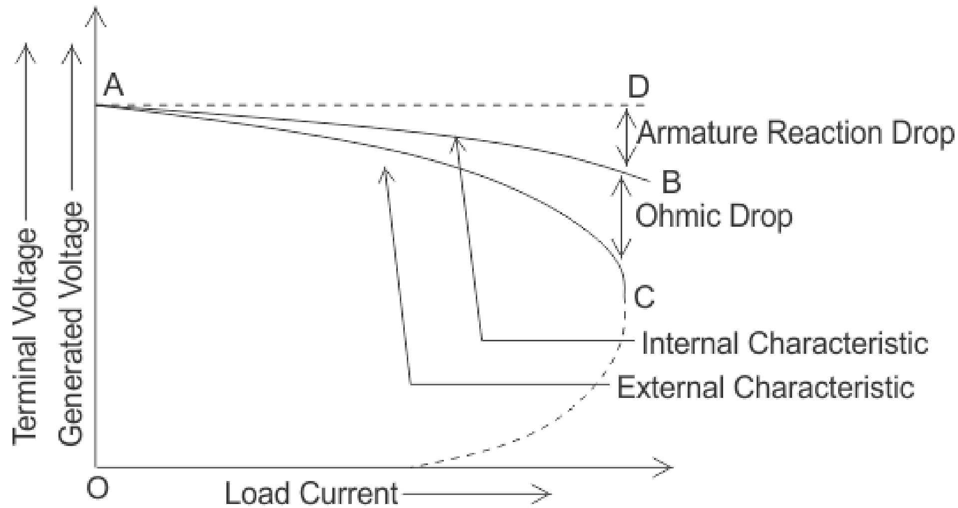
The internal characteristic curve represents the relation between the generated voltage E_g and the load current I_L. When the generator is loaded then the generated voltage is decreased due to armature reaction. So, generated voltage will be lower than the emf generated at no load. Here in the figure below AD curve is showing the no load voltage curve and AB is the internal characteristic curve.

External Characteristic of Shunt Wound DC Generator

AC curve is showing the external characteristic of the shunt wound DC generator. It is showing the variation of terminal voltage with the load current. Ohmic drop due to armature resistance gives lesser terminal voltage the generated voltage. That is why the curve lies below the internal characteristic curve.

$$Terminal\ voltage\ V = (E_g - I_a R_a) = E_g - (I_{sh} + I_L)R_a$$

The terminal voltage can always be maintained constant by adjusting the of the load terminal.



When the load resistance of a shunt wound DC generator is decreased, then load current of the generator increased as shown in above figure. But the load current can be increased to a certain limit with (upto point C) the decrease of load resistance. Beyond this point, it shows a reversal in the characteristic. Any decrease of load resistance, results in current reduction and consequently, the external characteristic curve turns back as shown in the dotted line and ultimately the terminal voltage becomes zero. Though there is some voltage due to residual magnetism. We know, Terminal voltage.

$$V = E_g - (I_{sh} + I_L)R_a$$

Now, when I_L increased, then terminal voltage decreased. After a certain limit, due to heavy load current and increased holmic drop, the terminal voltage is reduced drastically. This drastic reduction of terminal voltage across the load, results the drop in the load current although hat that time load is high or load resistance is low.That is why the load resistance of the machine must be maintained properly. The point in which the machine gives maximum current output is called breakdown point (point C in the picture).Magnetization Curve of DC Generator
DC generator is that curve which gives the relation between field current and the armature terminal voltage on open circuit.When the DC generator is driven by a prime mover then an emf is induced in the armature. The generated emf in the armature is given by an expression

$$E_g = \varphi P \frac{N}{60} \times \frac{Z}{A} volts = \varphi N \left(\frac{P}{60} \times \frac{Z}{A}\right) \quad or, \qquad E_g = K\varphi N$$

$$\left(\frac{P}{60} \times \frac{Z}{A}\right)$$

is constant for a given machine.it is replaced by K in this equation.
Here,

φ is the flux per pole,
P is the no. of poles,
N is the no. of revolution made by armature per minute,
Z is the no. of armature conductors,
A is no. of parallel paths.

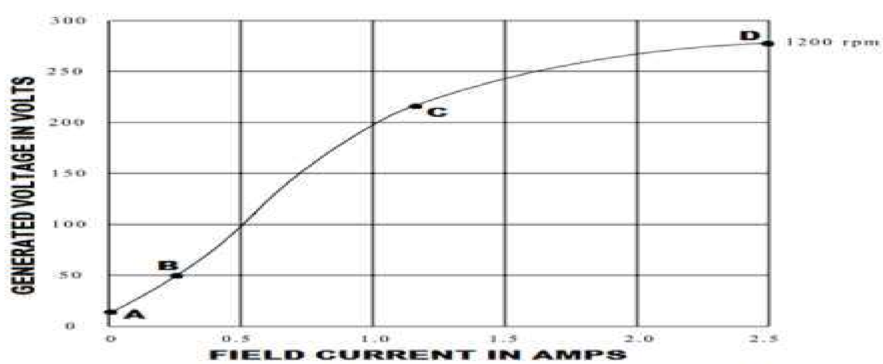
Now, from the equation we can clearly see that the generated emf is directly proportional to the product of flux per pole and the speed of the armature.

$E_g \propto \varphi N$

If the speed is constant, then the generated emf is directly proportional to the flux per pole.

$E_g \propto \varphi$ (N is constant)

It is obvious that, as the excitation current or field current (I_f) increases from its initial value, the flux and hence generated emf is increased with the field current. If we plot the generated voltage on the Y axis and field current on the X axis then themagnetization curve will be as shown in figure below.



MAGNATIZATION CURVE OF A DC GENERATOR

Magnetization curve of a DC generator has a great importance because it represents the saturation of the magnetic circuit. For this reason this curve is also called saturation curve.
According to the molecular theory of magnetism the molecules of a magnetic material, which is not magnetized, are not arranged or aligned in definite order. When current passed through the magnetic material then its molecules are arranged in definite order. Up to a certain value of field current the maximum molecules are arranged. In this stage the flux established in the pole increased directly with the field current and the generated voltage is also increased. Here, in this curve, point B to point C is showing this phenomena and this portion of the magnetization curve is almost a straight line. Above a certain point (point C in this curve) the nu-magnetized molecules become very fewer and it became very difficult to further increase in pole flux.This

point is called saturation point. Point C is also known as the knee of the magnetization curve. A small increase in magnetism requires very large field current above the saturation point. That is why upper portion of the curve (point C to point D) is bend as shown in figure.

Magnetization curve of a DC generator does not start from zero initially. It starts from a value of generated voltage due to residual magnetism.

Residual Magnetism

In ferromagnetic materials, the magnetic power and the generated voltage increase with the increase of the current flow through the coils. When current is reduced to zero, there is still magnetic power left in those coils core. This phenomenon is called residual magnetism. The core of a DC machine is made of ferromagnetic material.

Characteristics of Series Wound DC Generator

In these types of generators the field windings, armature windings and external load circuit all are connected in series as shown in figure below.

Series Wound Generator

Therefore, the same current flows through armature winding, field winding and the load.

Let, $I = I_a = I_{sc} = I_L$

Here, I_a = armature current

I_{sc} = series field current

I_L = load current

There are generally three most important characteristics of series wound DC generator which show the relation between various quantities such as series field current or excitation current, generated voltage, terminal voltage and load current.

Magnetic or Open Circuit Characteristic of Series Wound DC Generator

The curve which shows the relation between no load voltage and the field excitation current is called magnetic or open circuit characteristic curve. As during no load, the load terminals are open circuited, there will be no field current in the field since, the armature, field and load are series connected and these three make a closed loop of circuit. So, this curve can be obtained practically be separating the field winding and exciting the DC generator by an external source. Here in the diagram below AB curve is showing the magnetic characteristic of series wound DC generator. The linearity of the curve will continue till the saturation of the poles. After that there will be no further significant change of terminal voltage of DC generator for increasing field current. Due to residual magnetism there will be a small initial voltage across the armature that is why the curve started from a point A which is a little way up to the origin O.

Internal Characteristic of Series Wound DC Generator

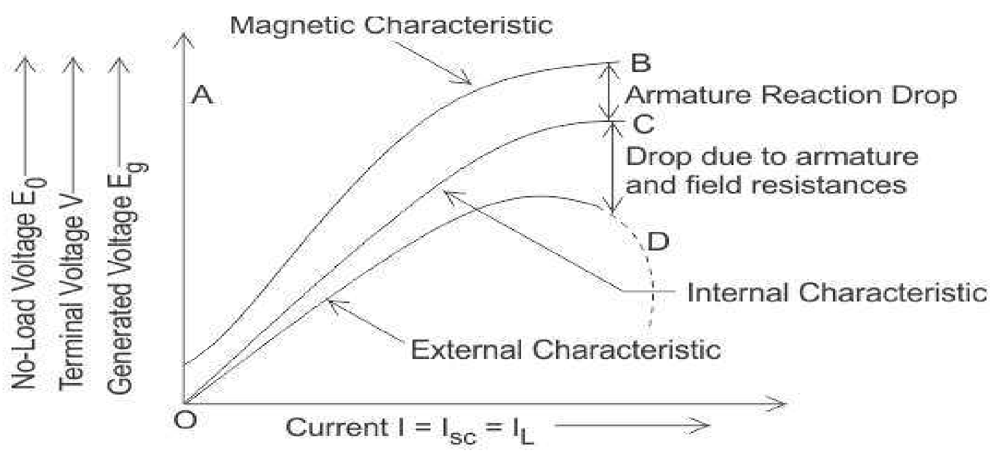
The internal characteristic curve gives the relation between voltage generated in the armature and the load current. This curve is obtained by subtracting the drop due to the demagnetizing effect of armature reaction from the no load voltage. So, the actual generated voltage (E_g) will be less than the no load voltage (E_0). That is why the curve is slightly dropping from the open circuit characteristic curve. Here in the diagram below OC curve is showing the internal characteristic or total characteristic of the series wound DC generator.

External Characteristic of Series Wound DC Generator

The external characteristic curve shows the variation of terminal voltage (V) with the load current (I_L). Terminal voltage of this type of generator is obtained by subtracting the ohomic drop due to armature resistance (R_a) and series field resistance (R_{sc}) from the actually generated voltage (E_g).

Terminal voltage $V = E_g - I(R_a + R_{sc})$

The external characteristic curve lies below the internal characteristic curve because the value of terminal voltage is less than the generated voltage. Here in the figure OD curve is showing the external characteristic of the series wound DC generator.

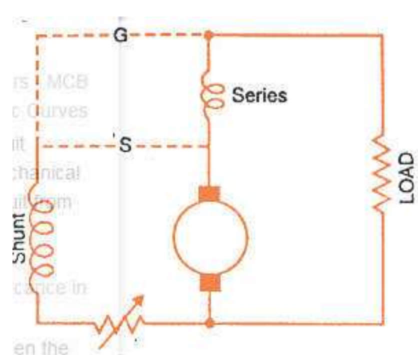


Characteristic Curves of Series Wound DC Generators

it can be observed from the characteristics of series wound DC generator, that with the increase in load (load is increased when load current increases) the terminal voltage of the machine increases. But after reaching its maximum value it starts to decrease due to excessive demagnetizing effect of armature reaction. This phenomenon is shown in the figure by the dotted line. Dotted portion of the characteristic gives approximately constant current irrespective of the external load resistance. This is because if load is increased, the field current is increased as field is series connected with load. Similarly if load is increased, armature current is increased as the armature is also series connected with load.

But due to saturation, there will be no further significance raise of magnetic field strength hence any further increase in induced voltage. But due to increased armature current, the affect of armature reaction increases significantly which causes significant fall in load voltage. If load voltage falls, the load current is also decreased proportionally since current is proportional to voltage as per Ohm’s law . So, increasing load, tends to increase the load current, but decreasing load voltage, tends to decrease load current. Due these two simultaneous effects, there will be no significant change in load current in dotted portion of external characteristics of series wound DC generator. That is why series DC generator is called constant current DC generator.

Characteristics of DC Compound Generator



In a compound generator, both series and shunt excitation are combined as shown in figure. The shunt winding can be connected either across the armature only (short-shunt connection S) or across armature plus series field (long-shunt connection G).

The compound generator can be cumulatively compounded or differentially compounded generator. The latter is rarely used in practice. Therefore, we shall discuss the characteristics of cumulatively compounded generator. It may be noted that external characteristics of long and short shunt compound generators are almost identical.

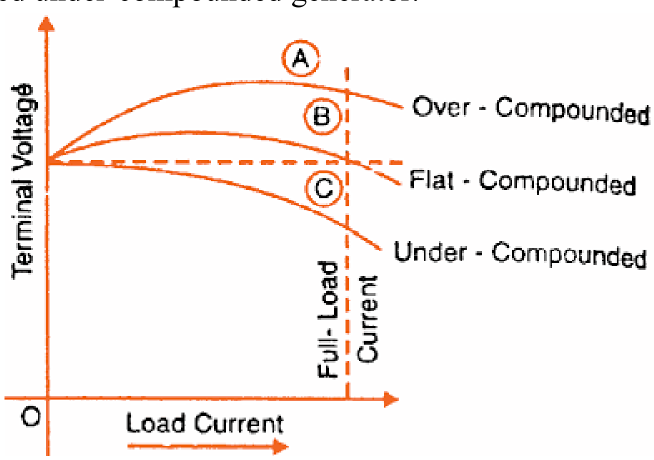
External characteristics

The external characteristics of a cumulatively compounded generator is shown in the figure. The series excitation aids the shunt excitation. The degree of compounding depends upon the increase in series excitation with the increase in load current.

- If series winding turns are so adjusted that with the increase in load current the terminal voltage increases, it is called over-compounded generator. In such a case, as the load current increases, the series field m.m.f. increases and tends to increase the flux and hence the generated voltage. The increase in generated voltage is greater than the IaRa drop so that instead of decreasing, the terminal voltage increases as shown by curve A.

- If series winding turns are so adjusted that with the increase in load current, the terminal voltage substantially remains constant, it is called flat-compounded generator. The series winding of such a machine has lesser number of turns than the one in over-compounded machine and, therefore, does not increase the flux as much for a given load current. Consequently, the full-load voltage is nearly equal to the no-load voltage as indicated by curve B.

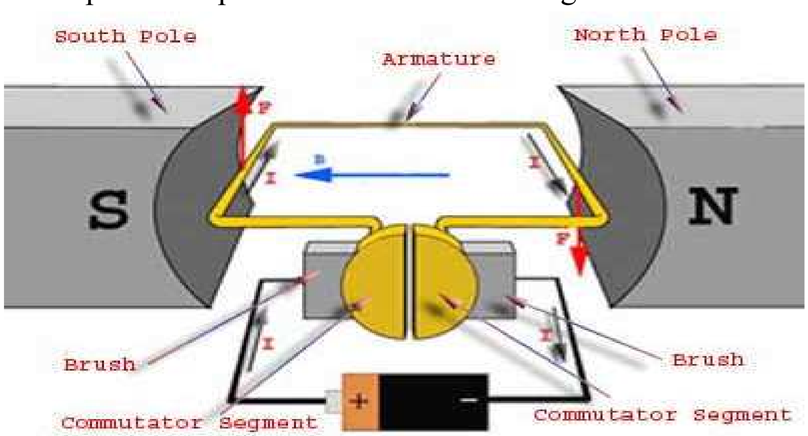
- If series field winding has lesser number of turns than for a flat compounded machine, the terminal voltage falls with increase in load current as indicated by curve C. Such a machine is called under-compounded generator.



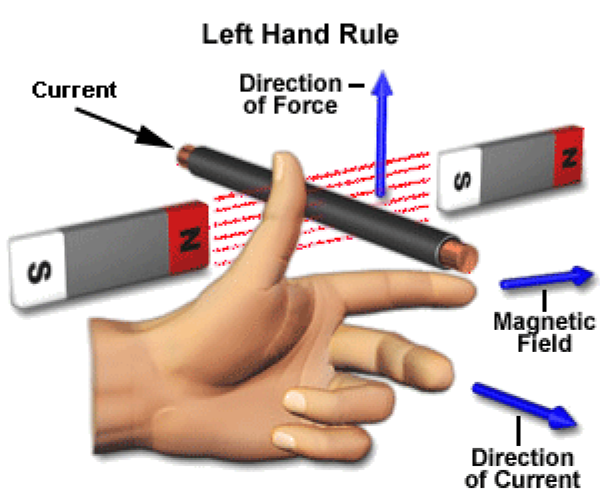
Working or Operating Principle of DC Motor

A DC motor in simple words is a device that converts electrical energy (direct current system) into mechanical energy. It is of vital importance for the industry today, and is equally important for engineers to look into the working principle of DC motor in details that has been discussed in this article. In order to understand the operating principle of DC motor we need to first look into its constructional feature.

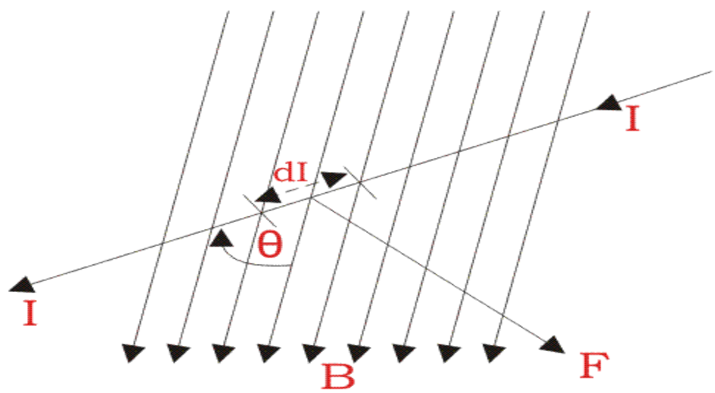
The very basic construction of a DC motor contains a current carrying armature which is connected to the supply end through commutator segments and brushes. The armature is placed in between north south poles of a permanent or an electromagnet as shown in the diagram above.



As soon as we supply direct current in the armature, a mechanical force acts on it due to electromagnetic effect of the magnet. Now to go into the details of the operating principle of DC motor its important that we have a clear understanding of Fleming's left hand rule to determine the direction of force acting on the armature conductors of DC motor.



If a current carrying conductor is placed in a magnetic field perpendicularly, then the conductor experiences a force in the direction mutually perpendicular to both the direction of field and the current carrying conductor. Fleming's left hand rule says that if we extend the index finger, middle finger and thumb of our left hand perpendicular to each other, in such a way that the middle finger is along the direction of current in the conductor, and index finger is along the direction of magnetic field i.e. north to south pole, then thumb indicates the direction of created mechanical force. For clear understanding the principle of DC motor we have to determine the magnitude of the force, by considering the diagram below.



We know that when an infinitely small charge dq is made to flow at a velocity 'v' under the influence of an electric field E, and a magnetic field B, then the Lorentz Force dF experienced by the charge is given by:-

$$dF = dq(E + vB)$$

For the **operation of DC motor**, considering E = 0

$$dF = dq \times v \times B$$

i.e. it's the cross product of dq v and magnetic field B.

$$dF = dq\frac{dL}{dt} \times B \qquad \left[V = \frac{dL}{dt}\right]$$

Where, dL is the length of the conductor carrying charge q.

$$dF = dq\frac{dL}{dt} \times B$$

$$or,\ dF = IdL \times B \qquad \left[Since,\ current\ I = \frac{dq}{dt}\right]$$

$$or,\ F = IL \times B = ILB\sin\theta$$

$$or,\ F = BIL\sin\theta$$

From the 1st diagram we can see that the construction of a DC motor is such that the direction of current through the armature conductor at all instance is perpendicular to the field. Hence the force acts on the armature conductor in the direction perpendicular to the both uniform field and current is constant.

$$i.e.\ \theta = 90^\circ$$

So if we take the current in the left hand side of the armature conductor to be I, and current at right hand side of the armature conductor to be -I, because they are flowing in the opposite direction with respect to each other.

Then the force on the left hand side armature conductor,

$$F_i = BIL \sin 90^\circ = BIL$$

Similarly force on the right hand side conductor

$$F_r = B(-I)L \sin 90^\circ = -BIL$$

Therefore, we can see that at that position the force on either side is equal in magnitude but opposite in direction. And since the two conductors are separated by some distance w = width of the armature turn, the two opposite forces produces a rotational force or a torque that results in the rotation of the armature conductor.

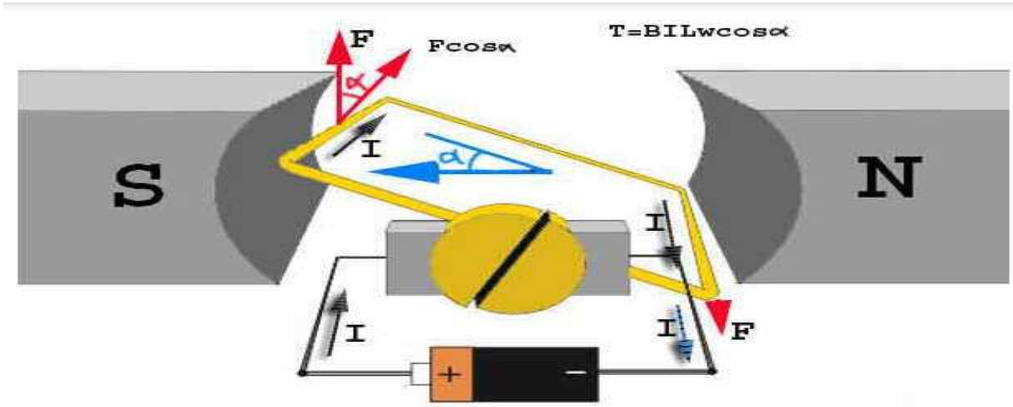
Now let's examine the expression of torque when the armature turn crate an angle of α (alpha) with its initial position.

The torque produced is given by,

$$Torque = (force,\ tangential\ to\ the\ direction\ of\ armature\ rotation) \times (distance)$$
$$or,\ \tau = F \cos\alpha \times w$$
$$or,\ \tau = BILw \cos\alpha$$

Where, α (alpha) is the angle between the plane of the armature turn and the plane of reference or the initial position of the armature which is here along the direction of magnetic field.

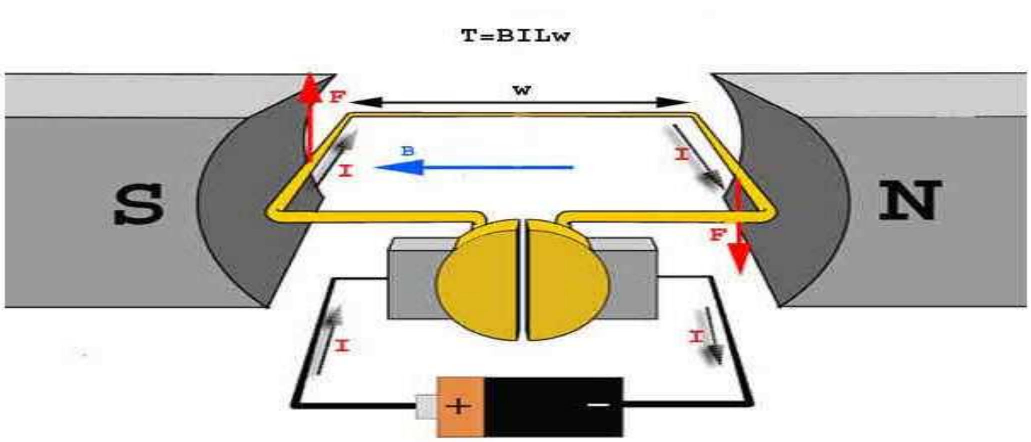
The presence of the term cosα in the torque equation very well signifies that unlike force the torque at all position is not the same. It in fact varies with the variation of the angle α (alpha). To explain the variation of torque and the principle behind rotation of the motor let us do a step wise analysis.



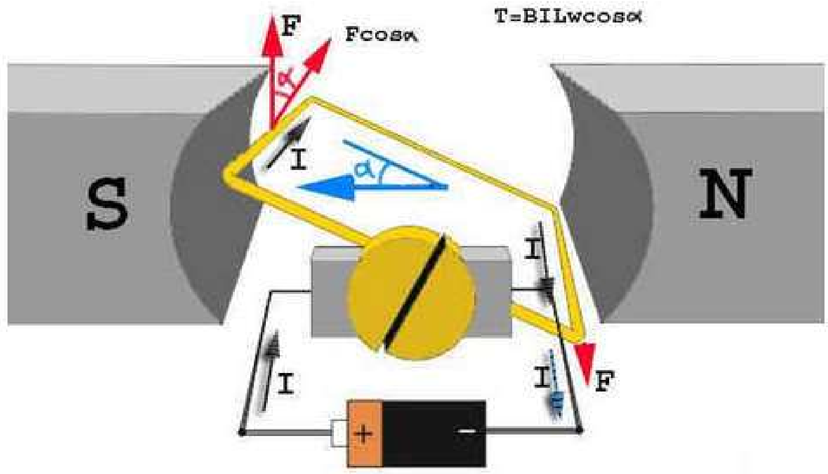
Step 1:
Initially considering the armature is in its starting point or reference position where the angle α = 0.

$$\therefore\ \tau = BILw \times \cos 0^\circ = BILw$$

Since, α = 0, the term cos α = 1, or the maximum value, hence torque at this position is maximum given by τ = BILw. This high starting torque helps in overcoming the initial inertia of rest of the armature and sets it into rotation.



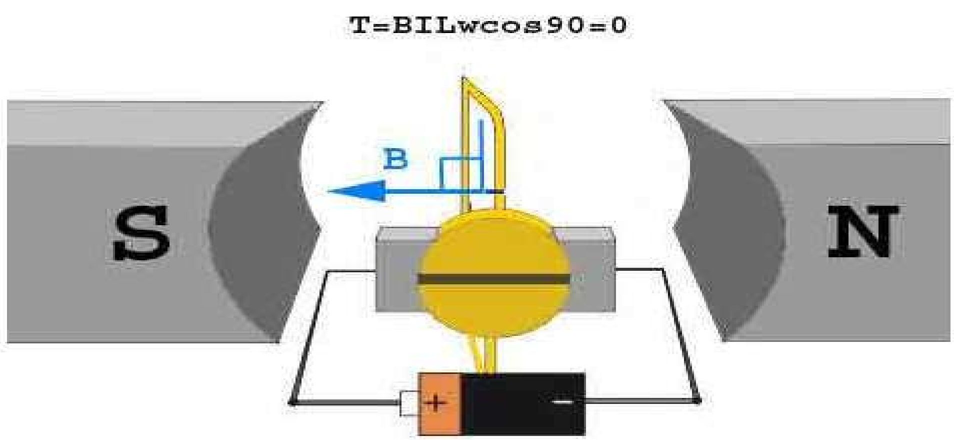
Step 2:
Once the armature is set in motion, the angle α between the actual position of the armature and its reference initial position goes on increasing in the path of its rotation until it becomes 90^o from its initial position. Consequently the term cosα decreases and also the value of torque.
The torque in this case is given by τ = BILwcosα which is less than BIL w when α is greater than 0^o.



Step 3:

In the path of the rotation of the armature a point is reached where the actual position of the rotor is exactly perpendicular to its initial position, i.e. $\alpha = 90^o$, and as a result the term $\cos\alpha = 0$. The torque acting on the conductor at this position is given by,

$$\therefore \tau = BIL\omega \times \cos 90^o = 0$$



i.e. virtually no rotating torque acts on the armature at this instance. But still the armature does not come to a standstill, this is because of the fact that the operation of DC motor has been engineered in such a way that the inertia of motion at this point is just enough to overcome this point of null torque. Once the rotor crosses over this position the angle between the actual position of the armature and the initial plane again decreases and torque starts acting on it again

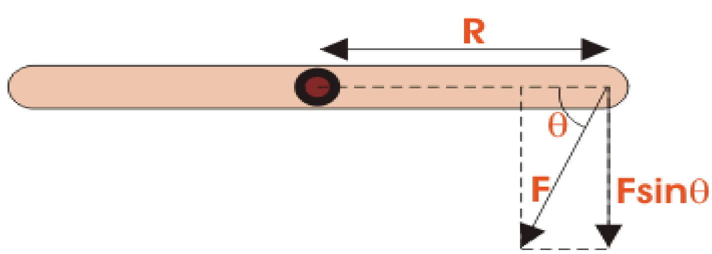
Torque Equation of DC Motor

When a DC machine is loaded either as a motor or as a generator, the rotor conductors carry current. These conductors lie in the magnetic field of the air gap. Thus each conductor experiences a force. The conductors lie near the surface of the rotor at a common radius from its center. Hence torque is produced at the circumference of the rotor and rotor starts rotating. The term torque as best explained by Dr. Huge d Young is the quantitative measure of the tendency of a force to cause a rotational motion, or to bring about a change in rotational motion. It is in fact the moment of a force that produces or changes a rotational motion.

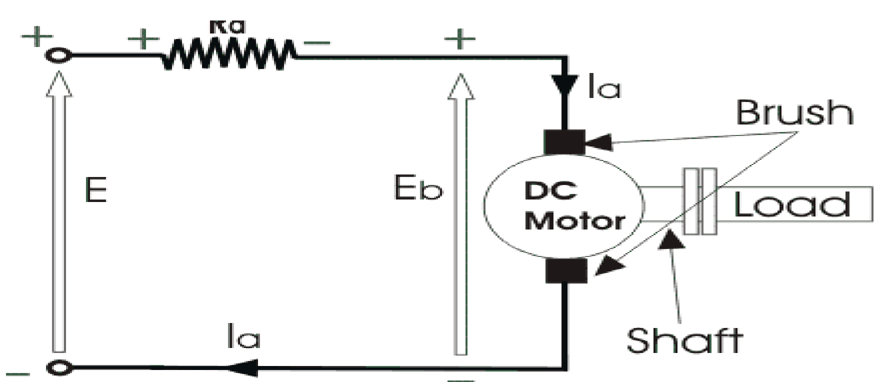
The equation of torque is given by,

$$\tau = FR\sin\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where, F is force in linear direction.
R is radius of the object being rotated,
and θ is the angle, the force F is making with R vector



The DC motor as we all know is a rotational machine, and torque of DC motor is a very important parameter in this concern, and it's of utmost importance to understand the torque equation of DC motor for establishing its running characteristics.
To establish the torque equation, let us first consider the basic circuit diagram of a DC motor, and its voltage equation.



Referring to the diagram beside, we can see, that if E is the supply voltage, E_b is the back emf produced and I_a, R_a are the armature current and armature resistance respectively then the voltage equation is given by,

$$E = E_b + I_a R_a \text{ .... } (2)$$

But keeping in mind that our purpose is to derive the torque equation of DC motor we multiply both sides of equation (2) by I_a.

$$Therefore,\ EI_a = E_b I_a + I_a^2 R_a \text{ .... } (3)$$

Now $I_a^2.R_a$ is the power loss due to heating of the armature coil, and the true effective mechanical power that is required to produce the desired torque of DC machine is given by,

$$P_m = E_b I_a \text{ .... } (4)$$

The mechanical power P_m is related to the electromagnetic torque T_g as,

$$P_m = T_g \omega \text{ .... } (5)$$

Where ω is speed in rad/sec.
Now equating equation (4) and (5) we get,

$$E_b I_a = T_g \omega$$

Now for simplifying the torque equation of DC motor we substitute.

$$E_b = \frac{P\varphi ZN}{60A} \text{ .... } (6)$$

Where, P is no of poles,
φ is flux per pole,
Z is no. of conductors,
A is no. of parallel paths,
and N is the speed of the DC motor.

$$Hence,\ w = \frac{2\pi N}{60} \text{ .... } (7)$$

Substituting equation (6) and (7) in equation (4), we get:

$$T_g = \frac{P.Z.\varphi.I_a}{2\pi A}$$

The torque we so obtain, is known as the electromagnetic torque of DC motor, and subtracting the mechanical and rotational losses from it we get the mechanical torque. Therefore,

$$T_m = T_g - mechanical\ losses$$

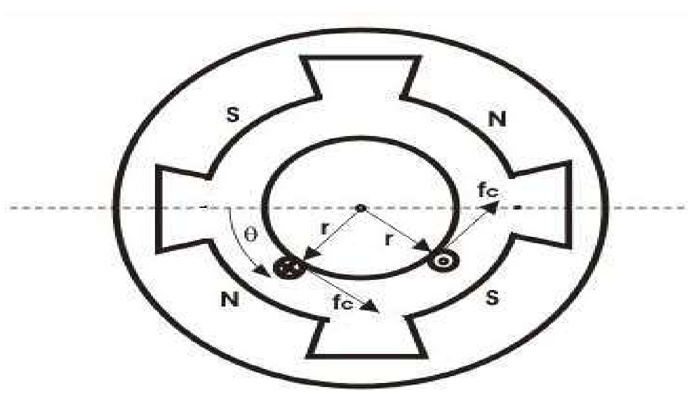
\This is the torque equation of DC motor. It can be further simplified as:

$$T_g = k_a \phi I_A$$

$$Where,\ k_a = \frac{P.Z}{2\pi A}$$

Which is constant for a particular machine and therefore the torque of DC motor varies with only flux φ and armature current I_a.

The Torque equation of a DC motor can also be explained considering the figure below.



$$Here\ we\ can\ see\ Area\ per\ pole\ A_r = \frac{2\pi.r.L}{P}$$

$$B = \frac{\varphi}{A_r}$$

$$Here\ we\ can\ see\ Area\ per\ pole\ A_r = \frac{2\pi.r.L}{P}$$

$$B = \frac{\varphi}{A_r}$$

$$B = \frac{P.\varphi}{2\pi r L}$$

Current/conductor $I_C = I_a$ A

Therefore, force per conductor = $f_C = BLI_a/A$

Now torque $T_C = f_C.\ r = BLI_a.r/A$

$$\therefore\ T_c = \frac{\varphi P.I_a}{2\pi A}$$

Hence, the total torque developed of a DC machine is,

$$T_g = \frac{P.Z.\varphi.I_a}{2\pi.A}$$

This torque equation of DC motor can be further simplified as:

$$T_g = k_a \phi I_a$$

$$Where,\ k_a = \frac{P.Z}{2\pi.A}$$

Which is constant for a particular machine and therefore the torque of DC motor varies with only flux φ and armature current I_a.

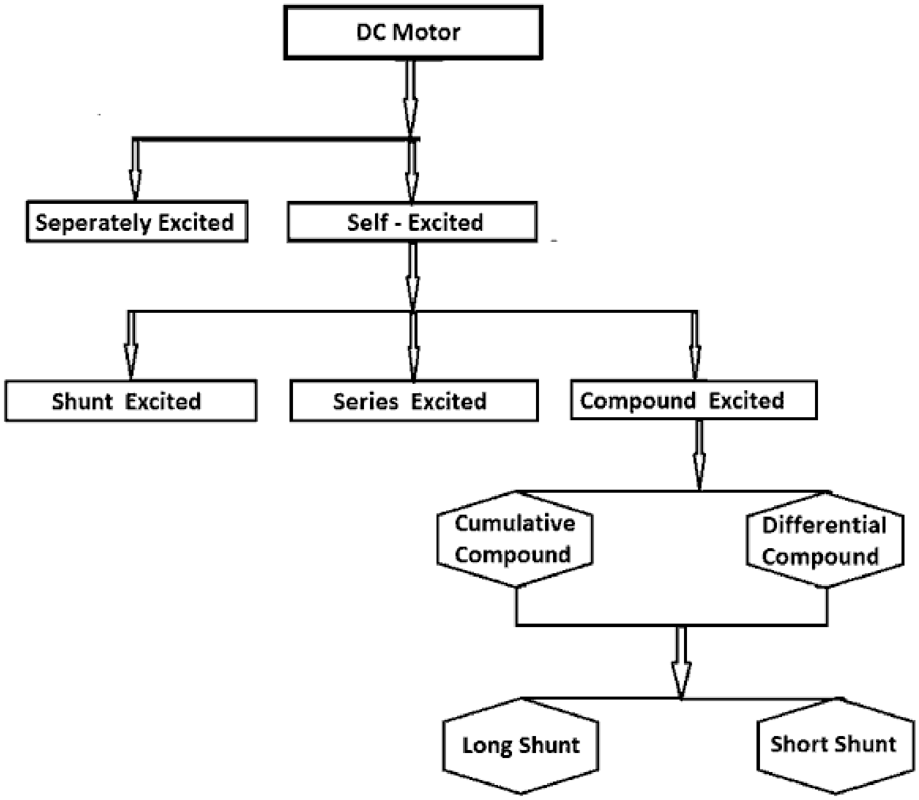
Types of DC motors

The direct current motor or the DC motor has a lot of application in today's field of engineering and technology. Starting from an electric shaver to parts of automobiles, in all small or medium sized motoring applications DC motors come handy. And because of its wide range of application different functional types of DC motor are available in the market for specific requirements.

The types of DC motor can be listed as follows-

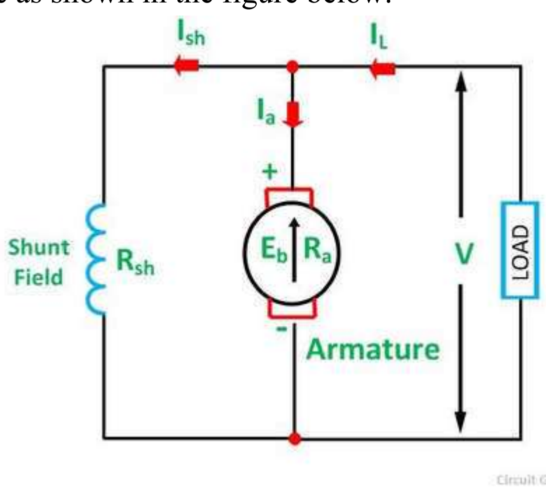
- Separately Excited DC Motor
- Shunt Wound DC Motor
- Series Wound DC Motor
- Compound Wound DC Motor
- Short shunt DC Motor

- Long shunt DC Motor



Shunt Wound Motor

This is the most common types of DC Motor. Here the field winding is connected in parallel with the armature as shown in the figure below.



Shunt Wound DC Motor

The current, voltage and power equations for a shunt motor are written as follows.

By applying KCL at the junction A in the above figure.

The sum of the incoming currents at A = Sum of the outgoing currents at A.

$$I = I_a + I_{sh} \dots\dots\dots (1)$$

Where,

I is the input line current
Ia is the armature current
Ish is the shunt field current

Equation (1) is the current equation.

The voltage equations are written by using Kirchhoff's voltage law (KVL) for the field winding circuit.

$$V = I_{sh}R_{sh} \dots\dots (2)$$

For armature winding circuit the equation will be given as

$$V = E + I_aR_a \dots\dots\dots (3)$$

The power equation is given as
Power input = mechanical power developed + losses in the armature + loss in the field.

$$VI = P_m + I_a^2R_a + I_{sh}^2R_{sh} \dots\dots\dots(4)$$

$$VI = P_m + I_a^2R_a + VI_{sh}$$

$$P_m = VI - VI_{sh} - I_a^2R_a = V(I - I_{sh}) - I_a^2R_a$$

$$P_m = VI_a - I_a^2R_a = (V - I_aR_a)I_a$$

$$P_m = EI_a \dots\dots\dots (5)$$

Multiplying equation (3) by Ia we get the following equations.

$$VI_a = EI_a + I_a^2R_a \dots\dots\dots (6)$$

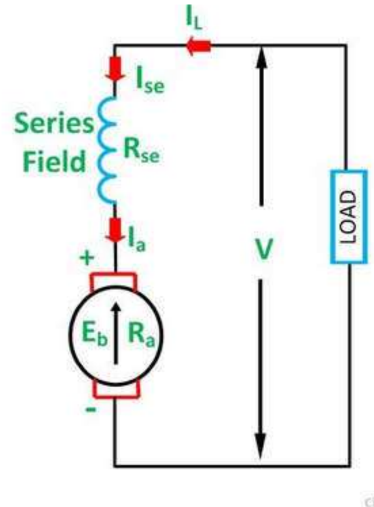
$$VI_a = P_m + I_a^2R_a \dots\dots\dots (7)$$

Where,

VIa is the electrical power supplied to the armature of the motor.

Series Wound Motor

In the series motor, the field winding is connected in series with the armature winding. The connection diagram is shown below.



Series Wound Motor

By applying the KCL in the above figure

Where, I_{se} is the series field current

The voltage equation can be obtained by applying KVL in the above figure

$$V = E + I\,(R_a + R_{se}) \dots \dots \dots (8)$$

The power equation is obtained by multiplying equation (8) by I we get

$$VI = EI + I^2\,(R_a + R_{se}) \dots \dots \dots (9)$$

Power input = mechanical power developed + losses in the armature + losses in the field

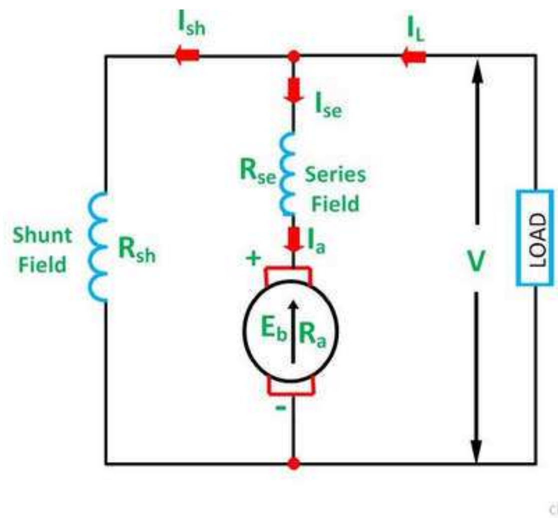
$$VI = P_m + I^2 R_a + I^2 R_a \dots \dots . . (10)$$

Comparing the equation (9) and (10), we will get the equation shown below.

$$P_m = EI \dots \dots \dots (11)$$

Compound Wound Motor

A DC Motor having both shunt and series field windings is called a Compound Motor. The connection diagram of the compound motor is shown below.



Compound Motor

The compound motor is further subdivided as Cumulative Compound Motor and Differential Compound Motor. In cumulative compound motor the flux produced by both the windings is in the same direction, i.e.

$$\varphi_r = \varphi_{sh} + \varphi_{se}$$

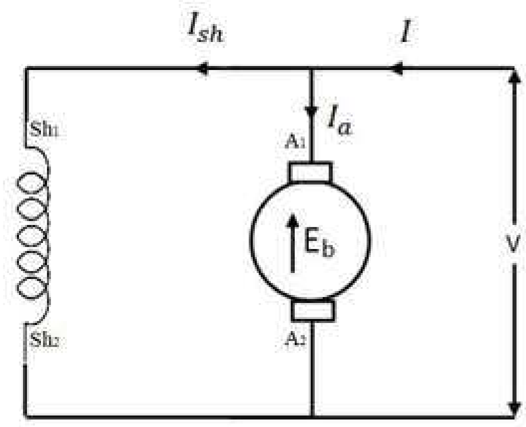
In differential compound motor, the flux produced by the series field windings is opposite to the flux produced by the shunt field winding, i.e.

$$\varphi_r = \varphi_{sh} - \varphi_{se}$$

The positive and negative sign indicates that direction of the flux produced in the field windings.

Back EMF

It is the dynamically induced emf in the armature conductors when the armature rotates following principle of DC motor. The direction of this induced emf can be determined using Fleming's right hand rule. This emf act in opposition to the supply voltage of the armature. It opposes the supply voltage that is why it is called back emf. The value of this induced emf is same as the value of the emf. induced in dc generator. The work done in overcoming this opposition is converted into mechanical energy.



Schematic diagram of DC shunt motor

DC shunt motor the rotating armature generating the back emf Eb. The armature current can be written as

$$I_a = \frac{V_T - E_b}{r_a}$$

Where ra is armature resistance,

$$E_b = \frac{P\phi ZN}{60A}$$

Armature current is proportional to back emf. So back emf is a controlling factor of armature current.

Characteristics of DC shunt motor

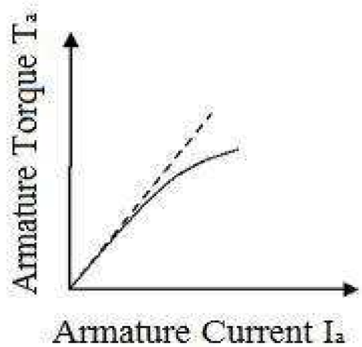
Armature torque vs armature current Tavs Ia characteristics

For a shunt motor flux can be assumed practically constant (at heavy loads, decreases, due to increased armature reaction)

$$T_a = k\phi I_a$$

K is constant, $T_a \propto I_a$

Therefore electrical characteristic of a shunt motor is a straight line through origin shown by dotted line in figure 3.3. Armature reaction weakens the flux hence Ta ys la characteristic bends as shown by dark line in figure 3.3, Shunt motors should never be started on heavy loads, since it draws heavy current under such condition.

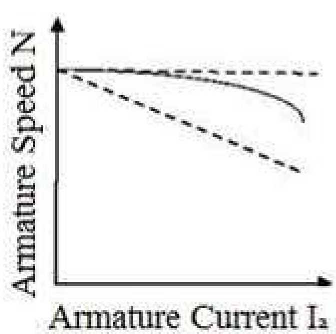


Torque Current Characteristic of DC shunt motor

$N \propto E_b / \phi$

$N = V - I_a r_a / \phi$

V is constant and in dc shunt motor is also constant. Thus with armature current speed drops and the speed current characteristics is drooping in nature is shown in figure



Speed vs armature torque Navs Ta characteristics

$N \propto E_b / \phi$

$N = V - I_a r_a / \phi$

characteristics of a DC series motor

Generally, three characteristic curves are considered important for DC motors which are, (i) Torque vs. armature current, (ii) Speed vs. armature current and (iii) Speed vs. torque. These are explained below for each type of Dc motor.These characteristics are determined by keeping the following two relations in mind.

$Ta \propto \phi.Ia$ and $N \propto E_b/\phi$

These above equations can be studied at - emf and torque equation. For a DC motor, magnitude of the back emf is given by the same emf equation of a dc generator i.e.

$Eb= P\phi NZ / 60A$. For a machine, P, Z and A are constant, therefore, $N \propto Eb/\phi$

Characteristics of DC Series Motors

Torque Vs. Armature Current (Ta-Ia)This characteristic is also known as electrical characteristic. We know that torque is directly proportional to the product of armature current and field flux,$Ta\propto \phi.Ia$. In DC series motors, field winding is connected in series with the armature, i.e. Ia= If. Therefore, before magnetic saturation of the field, flux ϕ is directly proportional to Ia. Hence, before magnetic saturation Ta α Ia2. Therefore, the Ta-Ia curve is parabola for smaller values of Ia. After magnetic saturation of the field poles, flux ϕ is independent of armature current Ia. Therefore, the torque varies proportionally to Ia only, $T \propto Ia$. Therefore, after magnetic saturation, Ta-Ia curve becomes a straight line. The shaft torque (Tsh) is less than armature torque (Ta) due to stray losses Hence, the curve Tsh vs Ia lies slightly lower.

In DC series motors, (prior to magnetic saturation) torque increases as the square of armature current, these motors are used where high starting torque is required.

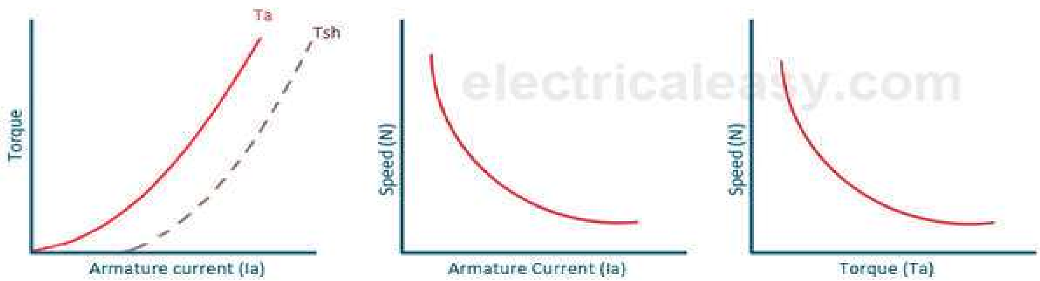
Speed Vs. Armature Current (N-Ia)

We know the relation, $N \propto Eb/\phi$.For small load current (and hence for small armature current) change in back emf Eb is small and it may be neglected. Hence, for small currents speed is inversely proportional to ϕ. As we know, flux is directly proportional to Ia, speed is inversely proportional to Ia. Therefore, when armature current is very small the speed becomes dangerously high. That is why a series motor should never be started without some mechanical load.

But, at heavy loads, armature current Ia is large. And hence, speed is low which results in decreased back emf Eb. Due to decreased Eb, more armature current is allowed.

Speed Vs. Torque (N-Ta)

This characteristic is also called as mechanical characteristic. From the above two characteristics of DC series motor, it can be found that when speed is high, torque is low and vice versa.



Characteristics of DC series motor

Characteristics of DC Compound Motor

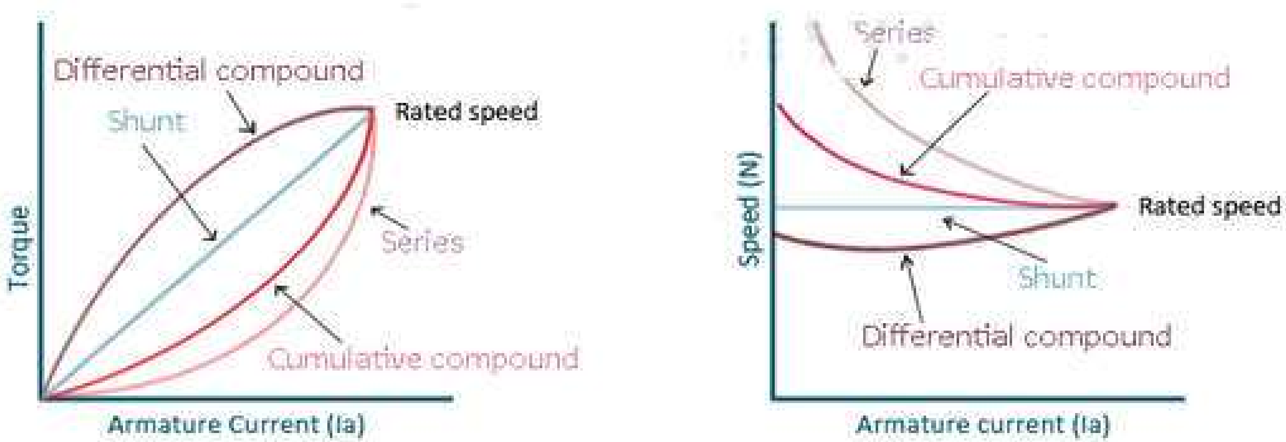
DC compound motors have both series as well as shunt winding. In a compound motor, if series and shunt windings are connected such that series flux is in direction as that of the shunt flux then the motor is said to be cumulatively compounded. And if the series flux is opposite to the direction of the shunt flux, then the motor is said to be differentially compounded. Characteristics of both these compound motors are explained below.

(a) Cumulative compound motor

Cumulative compound motors are used where series characteristics are required but the load is likely to be removed completely. Series winding takes care of the heavy load, whereas the shunt winding prevents the motor from running at dangerously high speed when the load is suddenly removed. These motors have generally employed a flywheel, where sudden and temporary loads are applied like in rolling mills.

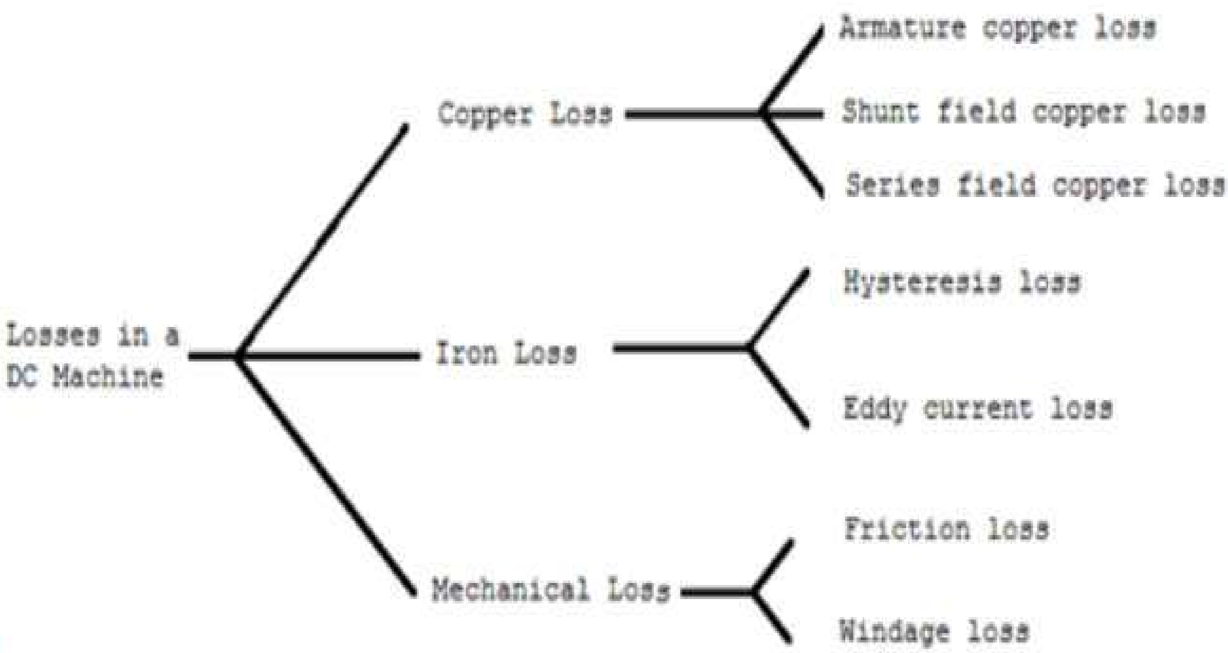
(b) Differential compound motor

` Since in differential field motors, series flux opposes shunt flux, the total flux decreases with increase in load. Due to this, the speed remains almost constant or even it may increase slightly with increase in load ($N \propto E_b/\phi$). Differential compound motors are not commonly used, but they find limited applications in experimental and research work.



Characteristics of DC compound motor

Types of Losses in a DC Machines



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The losses can be divided into three types in a dc machine (Generator or Motor). They are

1. Copper losses

2. Iron or core losses and
3. Mechanical losses.

All these losses seem as heat and therefore increase the temperature of the machine. Further the efficiency of the machine will reduce.

1. Copper Losses:

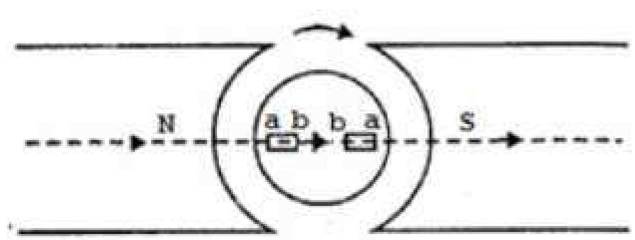
This loss generally occurs due to current in the various windings on of the machine. The different winding losses are;

Armature copper loss = I2a Ra
Shunt field copper loss = I2shRsh
Series field copper loss = I2se Rse
Note: There's additionally brush contact loss attributable to brush contact resistance (i.e., resistance in the middle of the surface of brush and commutator). This loss is mostly enclosed in armature copper loss.

2. Iron Losses

This loss occurs within the armature of a d.c. machine and are attributable to the rotation of armature within the magnetic field of the poles. They're of 2 sorts viz.,
(i) Hysteresis loss
(ii) eddy current loss.

Hysteresis loss



Hysteresis loss happens in the armature winding of the d.c. machine since any given part of the armature is exposed to magnetic field of reverses as it passes underneath sequence poles. The above fig shows the 2 pole DC machine of rotating armature. Consider a tiny low piece ab of the armature winding. Once the piece ab is underneath N-pole, the magnetic lines pass from a to b. Half a revolution well along, identical piece of iron is underneath S-pole and magnetic lines pass from b to a in order that magnetism within the iron is overturned. So as to reverse constantly the molecular magnets within the armature core, particular quantity of power must be spent that is named hysteresis loss. It's given by Steinmetz formula.

The steinmetz formula is

Hysteresis loss $P_h = \eta B_{16max} fV$ watts

Where,

η = Steinmetz hysteresis co-efficient

B_{max} = Maximum flux Density in armature winding

F = Frequency of magnetic reversals

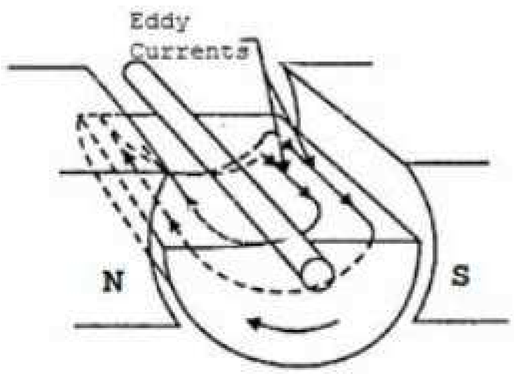
= NP/120 (N is in RPM)

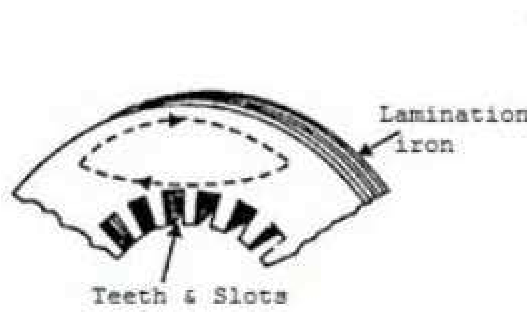
V = Volume of armature in m_3

If you want to cut back this loss in a d.c. machine, armature core is created of such materials that have an lesser value of Steinmetz hysteresis co-efficient e.g., silicon steel.

Eddy current loss:

In addition to the voltages evoked within the armature conductors, some of other voltages evoked within the armature core. These voltages turn out current currents within the coil core as shown in Fig. These are referred to as eddy currents and power loss attributable to their flow is named eddy current loss. This loss seems as heat that increases the temperature of the machine and efficiency will decrease.





If never-ending cast-iron core is employed, the resistance to eddy current path is tiny attributable to massive cross-sectional space of the core. Consequently, the magnitude of eddy current and therefore eddy current loss are massive. The magnitudes of eddy current are often decreased by creating core resistance as high as sensible. The core resistances are often greatly exaggerated by making the core of skinny, spherical iron sheets referred to as lamination's shown in the fig. The lamination's are insulated from one another with a layer of varnish. The insulating layer features a high resistance, thus only small amount of current flows from one lamination to the opposite. Also, as a result of every lamination is extremely skinny, the resistance to current passing over the breadth of a lamination is additionally quite massive. Therefore laminating a core will increase the core resistance that drops the eddy current and therefore the eddy current loss.

Eddy Current loss $P_e=K_eB^2_{max}f^2t^2V$ Watts

Where, k_e = constant

B_{max} = Maximum flux density in wb/m^2

T = Thickness of lamination in m

V = Volume of core in m3

Note: Constant (**Ke**) depend upon the resistance of core and system of unit used.

It may well be noted that eddy current loss be subject to upon the sq. of lamination thickness. For this reason, lamination thickness ought to be unbroken as tiny as potential.

*3.**Mechanical Loss***

These losses are attributable to friction and windage.

- Friction loss occurs due to the friction in bearing, brushes etc.

• windage loss occurs due to the air friction of rotating coil.

These losses rely on the speed of the machine. Except for a given speed, they're much constant.

Constant and Variable Losses

The losses in a d.c. machine is also further classified into (i) constant losses (ii) variable losses.

Constant losses

Those losses in a d.c. generator that stay constant at all loads are referred to as constant losses.

The constant losses in a very d.c. generator are:

(a)iron losses

(b)mechanical losses

(c)shunt field losses

Variable losses

Those losses in a d.c. generator that differ with load are referred to as variable losses. The variable losses in a very d.c. generator are:

Copper loss in armature winding (I2Ra)

Copper loss in series field winding (I2seRse)

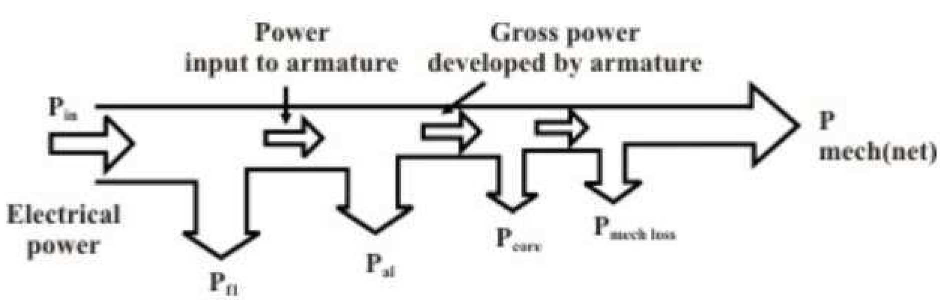
Total losses = Constant losses + Variable losses.

Generally this copper loss is constant for shunt and compound generators.

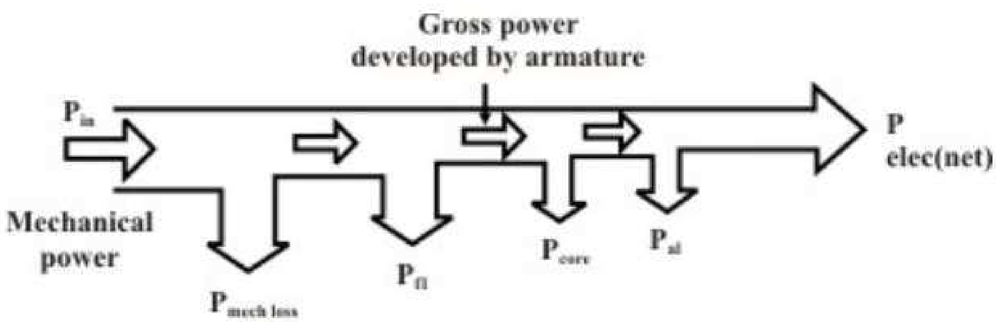
EFFICIENCY OF DC MOTOR

A portion of the input power is consumed by the field circuit as field copper loss. The remaining power is the power which goes to the armature; a portion of which is lost as core loss in the armature core and armature copper loss. Remaining power is the gross mechanical power developed of which a portion will be lost as friction and remaining power will be the net mechanical power developed. Obviously efficiency of the motor will be given by:

$$\eta = \frac{P_{\text{net mech}}}{P_{\text{in}}}$$

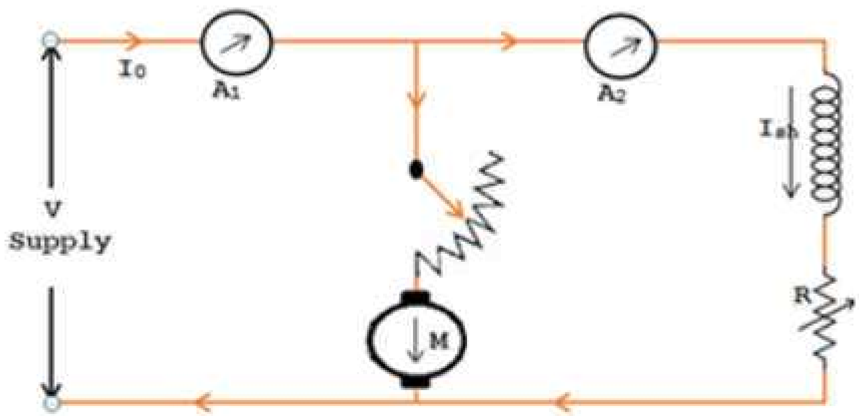


Similar power flow diagram of a d.c generator can be drawn to show various losses and input, output power



It is important to note that the name plate kW (or hp) rating of a d.c machine always corresponds to the net output at rated condition for both generator and motor.

Swinburne's Test for DC Machines



In this technique, the DC Generator or DC Motor is run as a motor at no load; with that losses of the DC machines are determined. When the losses of DC machine well-known, then we can find the efficiency of a DC machine in advance at any desired load. In DC machines this test is applicable only throughout the flux is constant at all load (DC Shunt machine and DC Compound Machine). This test maintains of two steps;

Determination of Hot Resistance of Windings:

The resistance of armature windings and shunt field windings are measured with the help of a battery, ammeter and voltmeter. Since these armature and shunt filed resistances are measured while the DC machine is cold, it should be transformed to values equivalent to the temperature at which the DC machine would work at full load. These values are measured generally when the room temperature increases above 40_oC. Take on the hot resistance of armature winding and shunt field winding be R_a and R_{sh} correspondingly.

Condition for maximum Efficiency in DC Machine

Determination of Constant Losses:

On no load the DC machine run as a motor with the supply voltage is varied to the normal rated voltage. With the use of the field regulator R the motor speed is varied to run the rated speed which is shown in the figure.

Let
V = Supply Voltage
I0 = No load current read by A1
Ish = Shunt Field current ready by A2
No load armature current Iao = I0 – Ish
No load Input power to motor = VI0
No load Input power to motor = VIa0 = V (I0 – Ish)

As the output power is nil, the no loads input power to the armature provides Iron loss, armature copper loss, friction loss and windage loss.

Constant loss Wc = Input power to Motor – Armature copper loss

Wc = VI0 – (I0 – Ish2Ra)

As the constant losses are identified, the efficiency of the DC machine at any loads can be determined. Suppose it is desired to determine the DC machine efficiency at no load current. Then,

Armature current Ia = I-Ish (For Motoring)
Ia = I+Ish (For Generating)

To find the Efficiency when running as a motor:

Input power to motor = VI
Armature copper loss =Ia2Ra = (I-Ish2Ra)
Constant Loss = Wc
Total Loss = (I-Ish2Ra)+Wc
Motor Efficiency η = (Input power – Losses)/ Input
η = {VI – (I-Ish2Ra)} / VI

Condition for maximum Efficiency in DC Machine

To find the Efficiency when running as a Generator:

Output Power of Generator = VI
Armature copper loss =Ia2Ra = (I+Ish2Ra)
Constant Loss = Wc
Total Loss = (I+Ish2Ra)+Wc
Motor Efficiency η= Output power/ (Output power + Losses)
η = VI / {VI + (I+Ish2Ra) + Wc}

Merits:

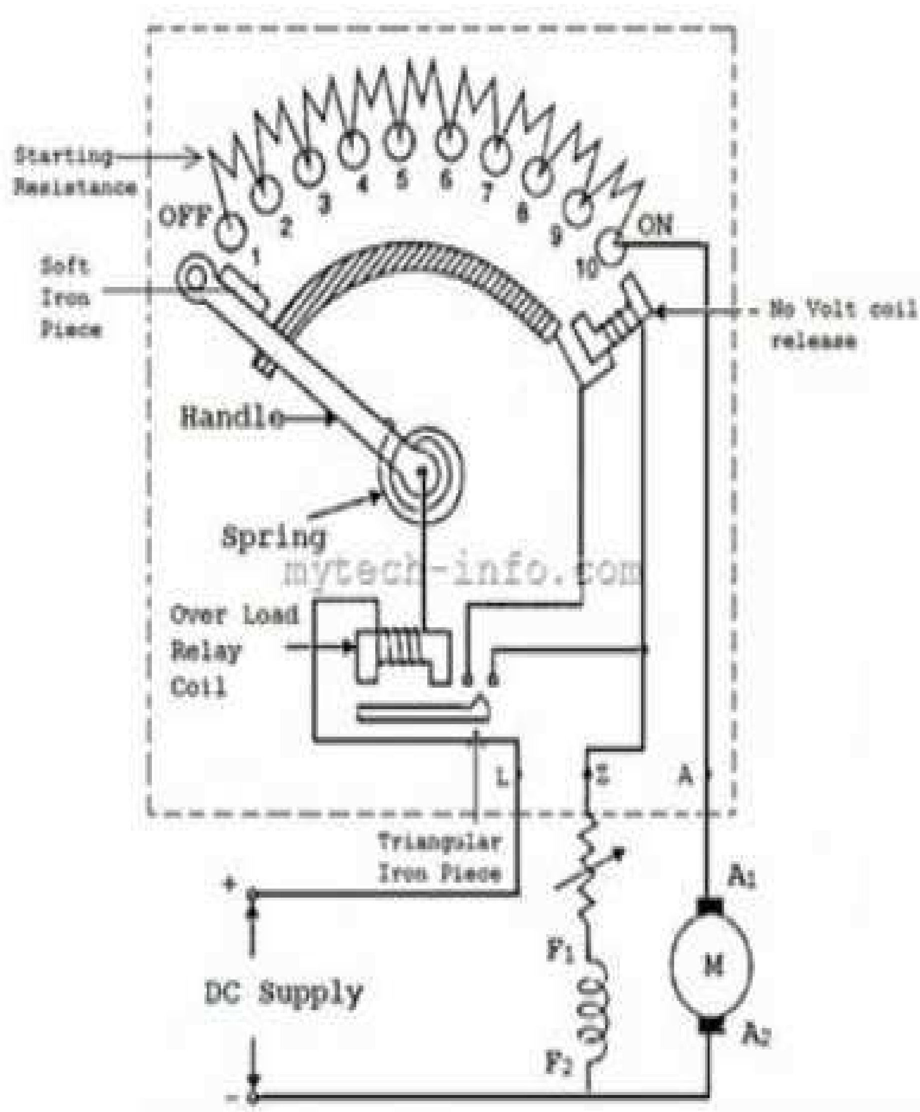
- Since this test is no load test, power required is less. Hence the cost is economic.
- The efficiency of the machine can be found very easily, because the constant losses are well known.
- This test is appropriate.

Demerits:

- When the DC machine is loaded, this test does not deliberate the stray load loss that occurs.
- Using this method we cannot check the DC machine performances at full load.

Three point starter

The figure below shows that typical representation diagram of a 3 point starter for DC shunt motors with its protective devices. It contains 3 terminals namely L, Z, & A; hence named 3 point starter. The starter is made up of starting resistances divided into many section and which are connected in series within the armature. The each tapping point on the starting resistances is carried out to a no. of studs. The starter 3 terminals L,Z & A are connected to the positive terminal of line, shunt field and armature terminal of motor respectively. The remaining terminal of the shunt and armature are connected to the negative line terminal. The No volt coil release is connected in series with field winding. The handle one end is connected to the L terminal by means of over load release coil. Then another end of handle travels against the twisting spring & make touching base with every single stud in the course of starting operation, tripping out the starting resistance as it moves above every stud in clockwise.



Speed control of DC motor

Many applications require the speed of a motor to be varied over a wide range. One of the most attractive features of DC motors in comparison with AC motors is the ease with which their speed can be varied.

We know that the back emf for a separately excited DC motor:

$$E_b = K\phi\omega_m = V_T - I_a R_a$$

Rearranging the terms,

$$\text{Speed } \omega_m = (V_T - I_a R_a)/K\phi$$

From the above equation, it is evident that the speed can be varied by using any of the following methods:

- Armature voltage control (By varying VT)
- Field Control (By Varying ϕ)
- Armature resistance control (By varying Ra)

Armature voltage control

This method is usually applicable to separately excited DC motors. In this method of speed control, Ra and ϕ are kept constant.
In normal operation, the drop across the armature resistance is small compared to Eb and therefore:

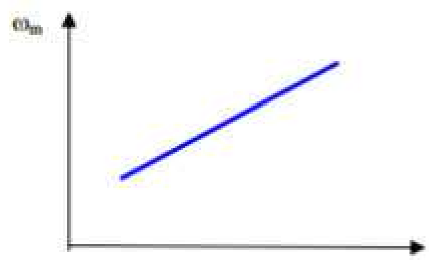
$$Eb \cong V_T$$

$$\text{Since, } Eb = K\text{ø}\ \omega_m$$

Angular speed can be expressed as:

$$\omega_m = V_T / K\text{ø}$$

From this equation, If flux is kept constant, the speed changes linearly with VT.
• As the terminal voltage is increased, the speed increases and vice versa.
• The relationship between speed and applied voltage is shown in figure 8. This method provides smooth variation of speed control.



.Variation of speed with applied voltage

Field Control , (ϕ)

In this method of speed control, Ra and VT remain fixed.

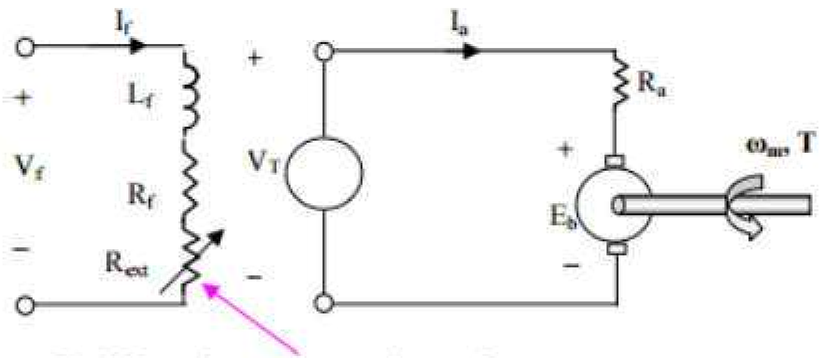
$$\omega_m \propto I/\phi$$

Assuming magnetic linearity, $\propto \phi$ If

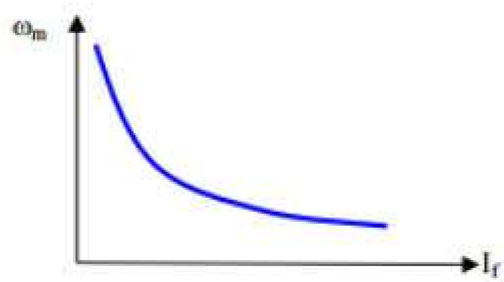
(OR) $\omega_m \propto I/I_F$

Speed can be controlled by varying field current If.

The field current can be changed by varying an adjustable rheostat in the field circuit • By increasing the value of total field resistance, field current can be reduced, and therefore speed can be increased.



The relationship between the field winding current and angular speed



Variation of speed with field current

Armature Resistance Control

The voltage across the armature can be varied by inserting a variable resistance in series with the armature circuit.

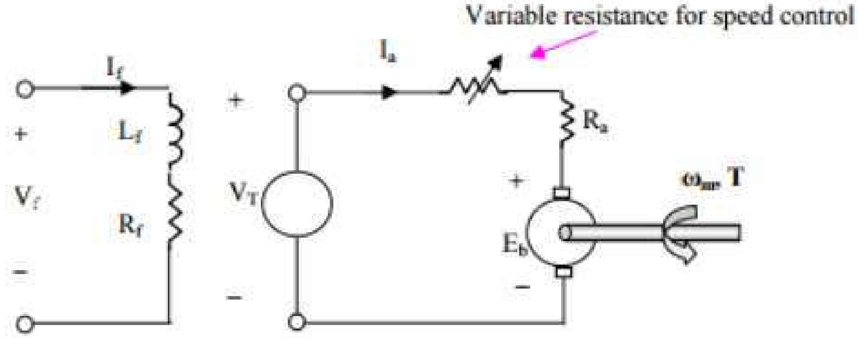
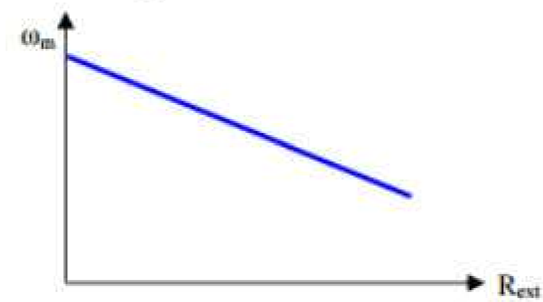


Fig.11. Armature resistance method for speed control

From speed-torque characteristics , we know that:

$$T_{dev} = \frac{K\phi}{R_a}(V_T - K\phi\omega_m)$$

For a load of constant torque V_T and ϕ are kept constant, as the armature resistance Ra is increased, speed decreases. As the actual resistance of the armature winding is fixed for a given motor, the overall resistance in the armature circuit can be increased by inserting an additional variable resistance in series with the armature. The variation if speed with respect to change in this external resistance is shown in figure 12. This method provides smooth control of speed



Variation of speed with external armature resistance

DC Shunt Motor speed control
All three methods described above can be used for controlling the speed of DC Shunt Motors.

Series Motor speed control
The speed is usually controlled by changing an external resistance in series with the armature. The other two methods described above are not applicable to DC series motor speed control.

UNIT-III

TRANSFORMERS & AC MACHINES

Introduction

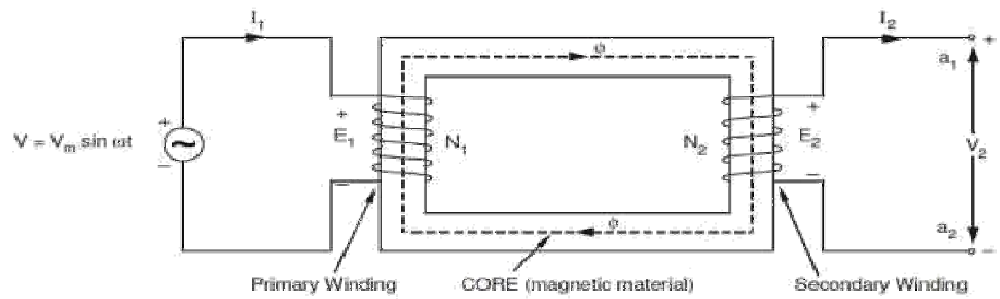
The transformer is a device that transfers electrical energy from one electrical circuit to another electrical circuit. The two circuits may be operating at different voltage levels but always work at the same frequency. Basically transformer is an electro-magnetic energy conversion device. It is commonly used in electrical power system and distribution systems. It can change the magnitude of alternating voltage or current from one value to another. This useful property of transformer is mainly responsible for the widespread use of alternating currents rather than direct currents i.e., electric power is generated, transmitted and distributed in the form of alternating current. Transformers have no moving parts, rugged and durable in construction, thus requiring very little attention. They also have a very high efficiency as high as 99%.

Single Phase Transformer

A transformer is a static device of equipment used either for raising or lowering the voltage of an a.c. supply with a corresponding decrease or increase in current. It essentially consists of two windings, the primary and secondary, wound on a common laminated magnetic core as shown in Fig 1. The winding connected to the a.c. source is called primary winding (or primary) and the one connected to load is called secondary winding (or secondary). The alternating voltage V1 whose magnitude is to be changed is applied to the primary.

Depending upon the number of turns of the primary (N1) and secondary (N2), an alternating e.m.f. E2 is induced in the secondary. This induced e.m.f. E2 in the secondary causes a secondary current I2. Consequently, terminal voltage V2 will appear across the load.

If V2 > V1, it is called a step up-transformer.
If V2 < V1, it is called a step-down transformer.



Constructional Details

Depending upon the manner in which the primary and secondary windings are placed on the core, and the shape of the core, there are two types of transformers, called

(a) core type, and

(b) shell type.

Core-type and Shell-type Construction

In core type transformers, the windings are placed in the form of concentric cylindrical coils placed around the vertical limbs of the core. The low-voltage (LV) as well as the high- voltage (HV) winding are made in two halves, and placed on the two limbs of core. The LV winding is placed next to the core for economy in insulation cost. Figure 2.1(a) shows the cross-section of the arrangement. In the shell type transformer, the primary and secondary windings are wound over the central limb of a three-limb core as shown in Figure 2.1(b). The HV and LV windings are split into a number of sections, and the sections are interleaved or sandwiched i.e. the sections of the HV and LV windings are placed alternately.

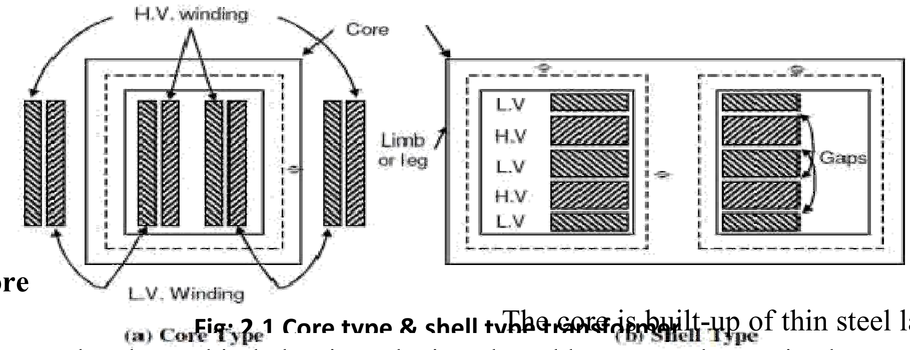


Fig: 2.1 Core type & shell type transformer

Core

The core is built-up of thin steel laminations insulated from each other. This helps in reducing the eddy current losses in the core, and also helps in construction of the transformer. The steel used for core is of high silicon content, sometimes heat treated to produce a high permeability and low hysteresis loss. The material commonly used for core is CRGO (Cold Rolled Grain Oriented) steel. Conductor material used for windings is mostly copper. However, for small distribution transformer aluminum is also sometimes used. The conductors, core and whole windings are insulated using various insulating materials depending upon the voltage.

Insulating Oil

In oil-immersed transformer, the iron core together with windings is immersed in insulating oil. The insulating oil provides better insulation, protects insulation from moisture and transfers the heat produced in core and windings to the atmosphere.

The transformer oil should possess the following qualities:

(a)High dielectric strength,
(b)Low viscosity and high purity,
(c)High flash point, and
(d)Free from sludge.

Transformer oil is generally a mineral oil obtained by fractional distillation of crude oil.

Tank and Conservator

The transformer tank contains core wound with windings and the insulating oil. In large transformers small expansion tank is also connected with main tank is known as conservator. Conservator provides space when insulating oil expands due to heating. The transformer tank is provided with tubes on the outside, to permits circulation of oil, which aides in cooling. Some additional devices like breather and Buchholz relay are connected with main tank. Buchholz relay is placed between main tank and conservator. It protect the transformer under extreme heating of transformer winding. Breather protects the insulating oil from moisture when the cool transformer sucks air inside. The silica gel filled breather absorbs moisture when air enters the tank. Some other necessary parts are connected with main tank like, Bushings, Cable Boxes, Temperature gauge, Oil gauge, Tapings, etc.

Principle of Operation

When an alternating voltage V1 is applied to the primary, an alternating flux ϕ is set up in the core. This alternating flux links both the windings and induces e.m.f.s E1 and E2 in them according to Faraday's laws of electromagnetic induction. The e.m.f. E1 is termed as primary e.m.f. and E2 is termed as secondary e.m.f.

Clearly, $$E_1 = -N_1 \frac{d\phi}{dt}$$

and $$E_2 = -N_2 \frac{d\phi}{dt}$$

$$\therefore \quad \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Note that magnitudes of E2 and E1 depend upon the number of turns on the secondary and primary respectively.

If N2 > N1, then E2 > E1 (or V2 > V1) and we get a step-up transformer. If N2 < N1, then
E2 < E1(or V2< V1) and we get a step-down transformer.

If load is connected across the secondary winding, the secondary e.m.f. E2 will cause a current I2 to flow through the load. Thus, a transformer enables us to transfer a.c. power from one circuit to another with a change in voltage level.

The following points may be noted carefully

(a)The transformer action is based on the laws of electromagnetic induction.

(b)There is no electrical connection between the primary and secondary.

(c)The a.c. power is transferred from primary to secondary through magnetic flux.

(d)There is no change in frequency i.e., output power has the same frequency as the input power.

(e)The losses that occur in a transformer are:

(a)core losses—eddy current and hysteresis losses

(b) copper losses—in the resistance of the windings

In practice, these losses are very small so that output power is nearly equal to the input primary power. In other words, a transformer has very high efficiency.

E.M.F. Equation of a Transformer

Consider that an alternating voltage V1 of frequency f is applied to the primary as shown in Fig.2.3. The sinusoidal flux ϕ produced by the primary can be represented as:

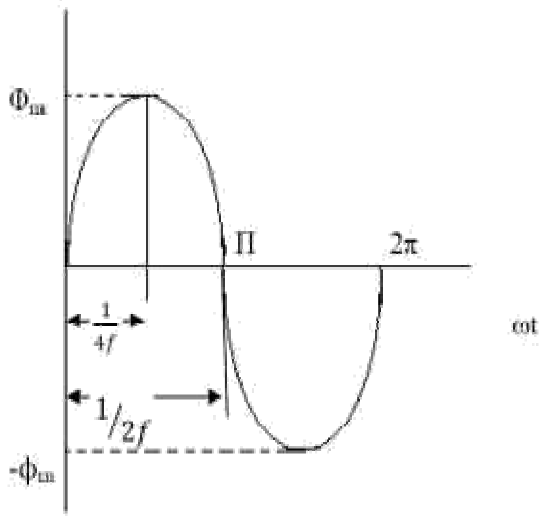
$$\phi=\phi m \sin\omega t$$

When the primary winding is excited by an alternating voltage V1, it is circulating alternating current, producing an alternating flux ϕ.

ϕ - Flux

ϕm - maximum value of flux ,N1 - Number of primary turns ,N2 - Number of secondary turns
f - Frequency of the supply voltage

E1 - R.M.S. value of the primary induced e.m.f ,E2 - R.M.S. value of the secondary induced e.m.f
The instantaneous e.m.f. e1 induced in the primary is –



The flux increases from zero value to maximum value ϕm in 1/4f of the time period that is in 1/4f seconds.
The change of flux that takes place in 1/4f seconds = ϕm - 0 = ϕm webers
Voltage Ratio

$$\frac{d\phi}{dt} = \frac{dt}{1/4f} = 4f\phi_m \ w_b/\text{sec.}$$

Since flux ϕ varies sinusoidally, the R.m.s value of the induced e.m.f is obtained by multiplying the average value with the form factor

$$\text{Form factor of a sinwave} = \frac{\text{R.m.s value}}{\text{Average value}} = 1.11$$

R.M.S Value of e.m.f induced in one turns = $4\phi_m f \times 1.11$ Volts.

= $4.44\phi_m f$ Volts.

R.M.S Value of e.m.f induced in primary winding = $4.44\phi_m f$ N1Volts.

R.M.S Value of e.m.f induced in secondary winding = $4.44\phi_m f$ N2Volts.

The expression of E_1 and E_2 are called e.m.f equation of a transformer

$$V_1 = E_1 = 4.44\phi_m f N_1 \text{Volts.}$$
$$V_2 = E_2 = 4.44\phi_m f N_2 \text{Volts.}$$

Voltage transformation ratio is the ratio of e.m.f induced in the secondary winding to the e.m.f induced in the primary winding.

$$\frac{E2}{E1} = \frac{4.44\phi mf\ N2}{4.44\phi mf\ N1}$$

$$\frac{E2}{E1} = \frac{N2}{N1} = K$$

This ratio of secondary induced e.m.f to primary induced e.m.f is known as voltage transformation ratio

$$E_2 = KE_1 \qquad \text{where } K = \frac{N2}{N1}$$

1. If N2>N1 i.e. K>1 we get E2>E1 then the transformer is called step up transformer.
2. If N2< N1 i.e. K<1 we get E2< E2 then the transformer is called step down transformer.
1. If N2= N1 i.e. K=1 we get E2= E2 then the transformer is called isolation transformer or 1:1 Transformer

Current Ratio

Current ratio is the ratio of current flow through the primary winding (I1) to the current flowing through the secondary winding (I2). In an ideal transformer -
Apparent input power = Apparent output power.
V1I1 = V2I2

$$\frac{I1}{I2} = \frac{V2}{V1} = \frac{N2}{N1} = K$$

Volt-Ampere Rating

i)The transformer rating is specified as the products of voltage and current (VA rating).
ii)On both sides, primary and secondary VA rating remains same. This rating is generally expressed in KVA (Kilo Volts Amperes rating)

$$\frac{V1}{V2} = \frac{I2}{I1} = K$$

$$V_1I_1 = V_2I_2$$

KVA Rating of a transformer $= \frac{V1I1}{1000} = \frac{V2I2}{1000}$ (1000 is to convert KVA to VA)

V_1 and V_2 are the V_t of primary and secondary by using KVA rating we can calculate I_1 and I_2 Full load current and it is safe maximum current.

I_1 Full load current $= \frac{KVA\ Rating\ X\ 1000}{V1}$

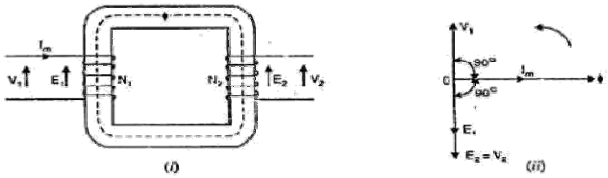
I_1 Full load current $= \frac{KVA\ Rating\ X\ 1000}{V2}$

Transformer on No-load

a) Ideal transformer
b) Practical transformer

a)Ideal Transformer

An ideal transformer is one that has
(i)No winding resistance
(ii)No leakage flux i.e., the same flux links both the windings
(iii)No iron losses (i.e., eddy current and hysteresis losses) in the core
Although ideal transformer cannot be physically realized, yet its study provides a very powerful tool inthe analysis of a practical transformer. In fact, practical transformers have properties that approach very close to an ideal transformer.



Consider an ideal transformer on no load i.e., secondary is open-circuited as shown in Fig.2.4 (i). under such conditions, the primary is simply a coil of pure inductance. When an alternating voltage V1 is applied to the primary, it draws a small magnetizing current Im which lags behind the applied voltage by 90°. This alternating current Im produces an alternating flux ϕ which is proportional to and in phase with it. The alternating flux ϕ links both the windings and induces

e.m.f. E1 in the primary and e.m.f. E2 in the secondary. The primary e.m.f. E1 is, at every instant, equal to and in opposition to V1 (Lenz's law). Both e.m.f.s E1 and E2 lag behind flux ϕ by 90°.However, their magnitudes depend upon the number of primary and secondary turns. Fig. 2.4 (ii) shows the phasor diagram of an ideal transformer on no load. Since flux ϕ is common to both the windings, it has been taken as the reference phasor. The primary e.m.f. E1 and secondary e.m.f. E2 lag behind the flux ϕ by 90°. Note that E1 and E2 are in phase. But E1 is equal to V1 and 180° out of phase with it.

$$\frac{E2}{E1} = \frac{V2}{V1} = K$$

Phasor Diagram

i)Φ (flux) is reference

ii)Im produce ϕ and it is in phase with ϕ, V1 Leads Im by 90˚

E1 and E2 are in phase and both opposing supply voltage V1, winding is purely inductive So current has to lag voltage by 90˚.

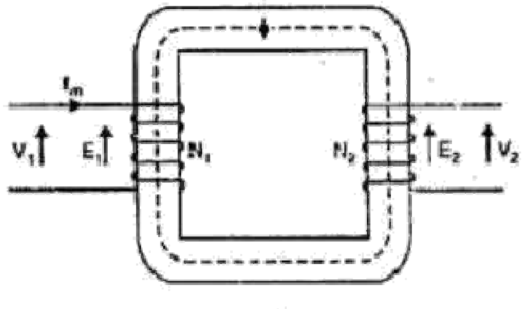
iii)The power input to the transformer

P = V1.I1. cos (90˚) ……….. (cos90˚ = 0) P= 0 (ideal transformer)

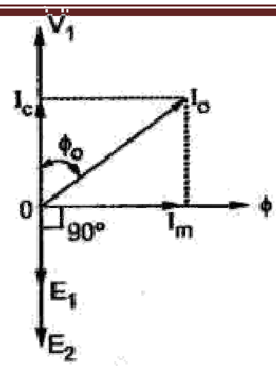
b)i) Practical Transformer on no load

A practical transformer differs from the ideal transformer in many respects. The practical transformer has (i) iron losses (ii) winding resistances and (iii) Magnetic leakage

(i)Iron losses. Since the iron core is subjected to alternating flux, there occurs eddy current and hysteresis loss in it. These two losses together are known as iron losses or core losses. The iron losses depend upon the supply frequency, maximum flux density in the core, volume of the core etc. It may be noted that magnitude of iron losses is quite small in a practical transformer.



(i)



(ii)

(ii)Winding resistances. Since the windings consist of copper conductors, it immediately follows that both primary and secondary will have winding resistance. The primary resistance R1 and secondary resistance R2 act in series with the respective windings as shown in Fig. When current flows through the windings, there will be power loss as well as a loss in voltage due to IR drop. This will affect the power factor and E1 will be less than V1 while V2 will be less than E2.
Here the primary will draw a small current I0 to supply -
(i)The iron losses and
(ii)A very small amount of copper loss in the primary.

Hence the primary no load current I0 is not 90° behind the applied voltage V1 but lags it by an angle ϕ0 < 90° as shown in the phasor diagram. No load input power, W0 = V1 I0 cos ϕ0
As seen from the phasor diagram in Fig.2.5 (ii), the no-load primary current I0
(i) The component Ic in phase with the applied voltage V1. This is known as active or working or iron loss component and supplies the iron loss and a very small primary copper loss.

$$Ic = I_0 \cos \phi_0$$

The component Im lagging behind V1 by 90° and is known as magnetizing component. It is this component which produces the mutual flux ϕ in the core.
$Im = I_0 \sin \phi_0$, I_0 phasor sum of I_M and I_C

$$I_o = \sqrt{Im^2 + Ic^2}$$

$$\text{No load P.F., } \cos\phi o = \frac{Ic}{Io}$$

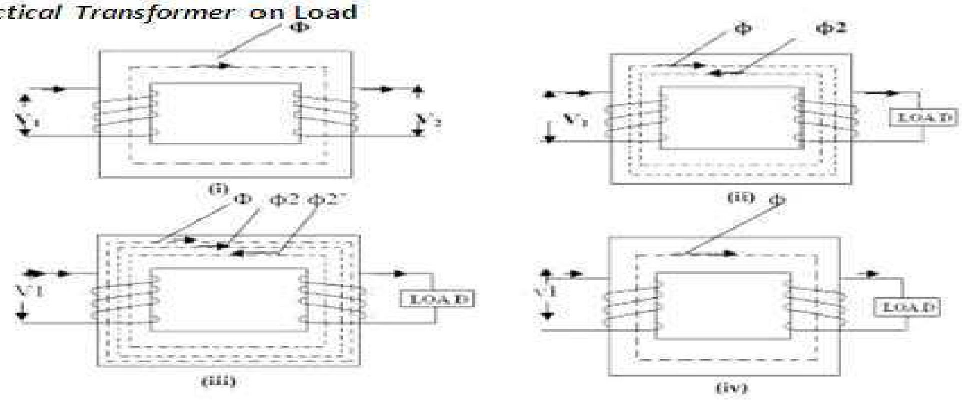
The no load primary copper loss (i.e. Io^2R1) is very small and may be neglected.

Therefore, the no load primary input power is practically equal to the iron loss in the transformer i.e., No load input power, W0 = V1Iocosϕo = Pi = Iron loss

At no load, there is no current in the secondary so that V2 = E2. On the primary side, the drops in R1 and X1, due to I0 are also very small because of the smallness of I0. Hence, we can say that at no load, V1 = E1.

i) When transformer is loaded, the secondary current I2 is flows through the secondary winding.

b) ii)Practical Transformer on Load



ii)Already Im magnetizing current flow in the primary winding fig

iii)The magnitude and phase of I2 with respect to V2 is determined by the characteristics of the load. a) I2 in phase with V2 (resistive load)

b)I2 lags with V2 (Inductive load)

c)I2 leads with V2 (capacitive load)

iv)Flow of secondary current I2 produce new Flux ϕ2 fig.

v)Φis main flux which is produced by the primary to maintain the transformer as constant magnetising component.

vi)Φ2 opposes the main flux ϕ, the total flux in the core reduced. It is called demagnetising Ampere- turns due to this E1 reduced.

vii)To maintain the ϕ constant primary winding draws more current (I2') from the supply (load component of primary) and produce ϕ2' flux which is oppose ϕ2 (but in same direction as ϕ), to maintain flux constant flux constant in the core fig.

viii)The load component current I2' always neutralizes the changes in the load.

ix)Whatever the load conditions, the net flux passing through the core is approximately the same as at no-load. An important deduction is that due to the constancy of core flux at all loads, the core loss is also practically the same under all load conditions fig.

$$\Phi_2 = \phi_2' \quad N_2 I_2 = N_1 I_2' \quad I_2' = \frac{N2}{N1} X I_2 = K I_2$$

Phasor Diagram

i)Take (ϕ) flux as reference for all load

ii)The load component I2', which is in anti-phase with I2 and phase of I2 is decided by the load.

iii)Primary current I1 is vector sum of Io and I2'

$$\vec{I_1} - \vec{I_o} + \vec{I_2'}$$

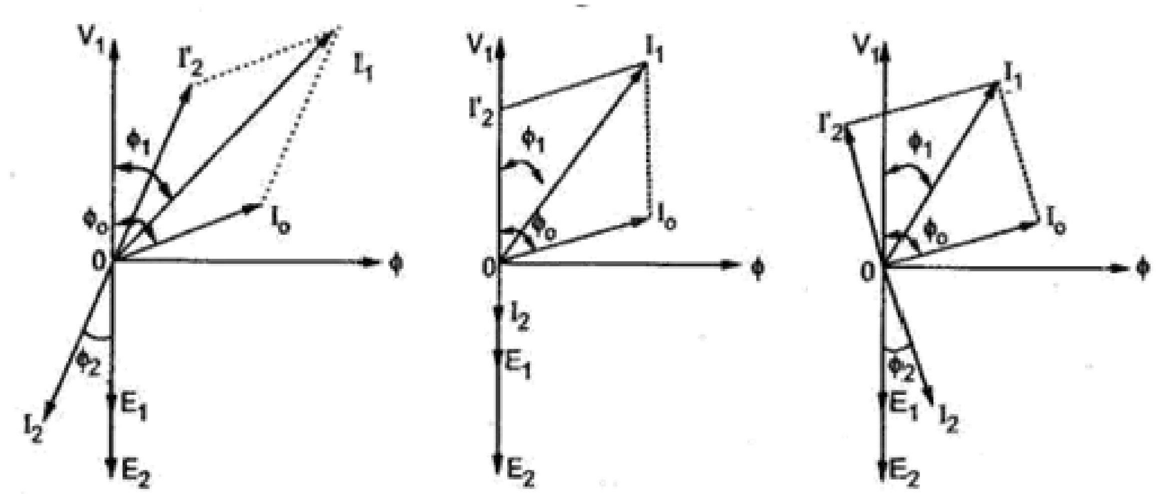
$$I_1 = \sqrt{Io^2 + I2'^2}$$

a)If load is Inductive, I2 lags E2 by ϕ2, shown in phasor diagram fig

b)If load is resistive, I2 in phase with E2 shown in phasor diagram fig.

c)If load is capacitive load, I2 leads E2 by ϕ2 shown in phasor diagram fig

For easy understanding at this stage here we assumed E2 is equal to V2 neglecting various drops.



(a) Inductive load (b) Resistive load (c) Capacitive load

$$\vec{I_1} = \vec{I_0} + \vec{I_2'}$$

$$I_1 \cong I_2' \qquad I_1 = \sqrt{Io^2 + I2'^2}$$

Balancing the ampere – turns

$$N_1 I_2' = N_1 I_1 + N_2 I_2$$

$$\frac{I1}{I2} = \frac{N2}{N1} = K$$

Now we going to construct complete phasor diagram of a transformer (shown in Fig: 2.7.b)

Effect of Winding Resistance

In practical transformer it process its own winding resistance causes power loss and also the voltage drop.

R1 – primary winding resistance in ohms. R2 – secondary winding resistance in ohms.

The current flow in primary winding make voltage drop across it is denoted as I1R1 here supply voltage V1 has to supply this drop primary induced e.m.f E1 is the vector difference between V1 and I1R1.

$$\vec{E_1} = \vec{V_1} - \vec{I_1 R_1}$$

Similarly the induced e.m.f in secondary E2, The flow of current in secondary winding makes voltage drop across it and it is denoted as I2R2 here E2 has to supply this drop.

The vector difference between E2 and I2R2

$$\vec{V_2} = \vec{E_2} - \vec{I_2 R_2}$$

(Assuming as purely resistive drop here)

Equivalent Resistance

1)It would now be shown that the resistances of the two windings can be transferred to any one of the two winding.

2)The advantage of concentrating both the resistances in one winding is that it makes calculations very simple and easy because one has then to work in one winding only.

3)Transfer to any one side either primary or secondary without affecting the performance of the transformer.

The total copper loss due to both the resistances

$$\text{Total copper loss} = I_1^2R_1 + I_2^2R_2$$
$$= I_1^2[R_1 + \frac{I_2^2}{I_1^2}]$$
$$= I_1^2[R_1 + \frac{1}{K^2}R_2]$$

$\frac{R2}{K^2}$ is the resistance value of R_2 shifted to primary side and denoted as R_2'.

R_2' is the equivalent resistance of secondary referred to primary

$$R_2' = \frac{R2}{K^2}$$

Equivalent resistance of transformer referred to primary fig (ii)

$$R_{1e} = R_1 + R_2' = R_1 + \frac{R2}{K^2}$$

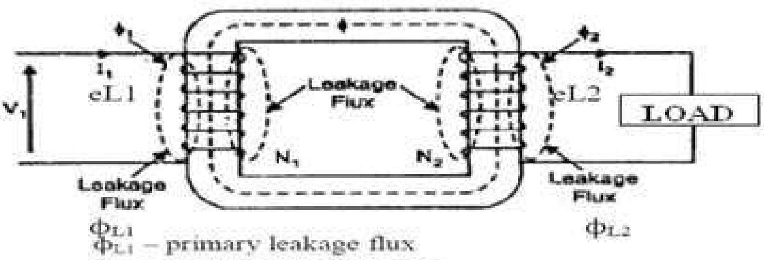
Effect of Leakage Reactance

i) It has been assumed that all the flux linked with primary winding also links the secondary winding. But, in practice, it is impossible to realize this condition.

- However, primary current would produce flux ɸwhich would not link the secondary winding. Similarly, current would produce some flux ɸ that would not link the primary winding.

- The flux ɸL1 complete its magnetic circuit by passing through air rather than around the core, as shown in fig.2.9. This flux is known as primary leakage flux and is proportional to the primary ampere – turns alone because the secondary turns do not links the magnetic circuit of ɸL1. It induces an e.m.f eL1 in primary but not in secondary.

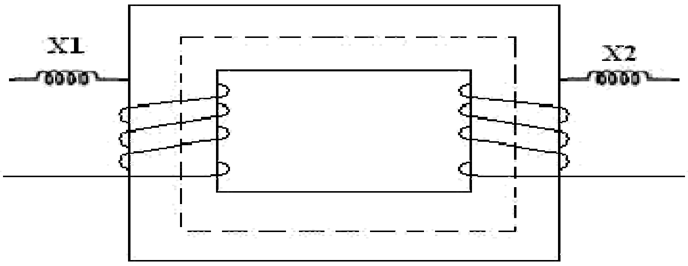
- The flux ɸL2 complete its magnetic circuit by passing through air rather than around the core, as shown in fig. This flux is known as secondary leakage flux and is proportional to the secondary ampere– turns alone because the primary turns do not links the magnetic circuit of ɸL2. It induces an e.m.f eL2 in secondary but not in primary.



ϕ_{L1} – primary leakage flux
ϕ_{L2} – secondary leakage flux
e_{L1} – self induced e.m.f (primary)
e_{L2} –self induced e.m.f (secondary)

Equivalent Leakage Reactance

Similarly to the resistance, the leakage reactance also can be transferred from primary to secondary. The relation through K^2 remains same for the transfer of reactance as it is studied



earlier for the resistance

X1 – leakage reactance of primary. X2 - leakage reactance of secondary.

Then the total leakage reactance referred to primary is X1e given by

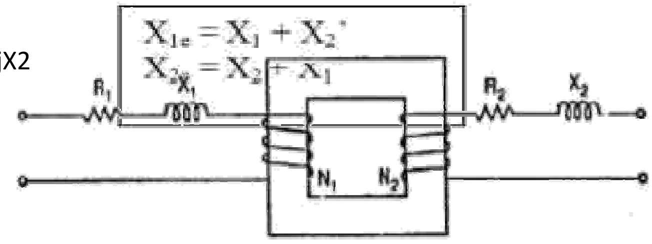
$$X_{1e} = X_1 + X_2'$$

$$X_2' = \frac{X2}{K^2}$$

The total leakage reactance referred to secondary is X_{2e} given by

$$X_{2e} = X_2 + X_1'$$
$$X_1' = K^2 X_1$$

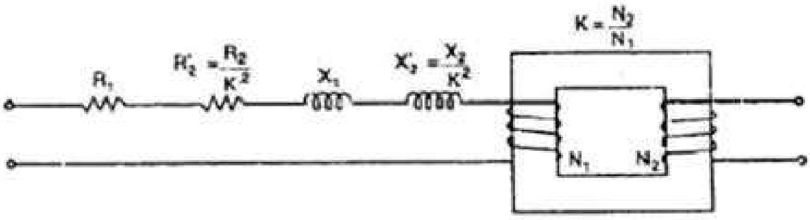
Z2 = R2 + jX2



Individual magnitude of Z1 and Z2 are

$$Z_1 = \sqrt{R1^2 + X1^2}$$
$$Z_2 = \sqrt{R2^2 + X2^2}$$

Similar to resistance and reactance, the impedance also can be referred to any one side,



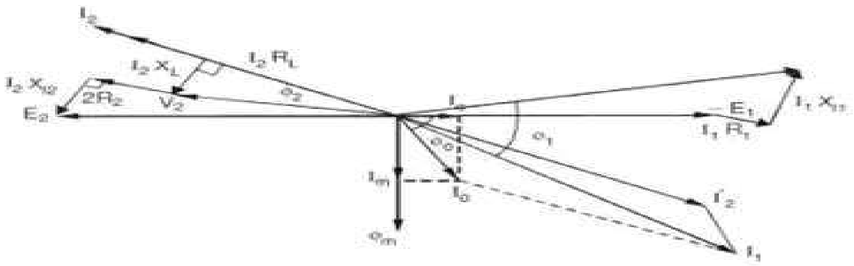
Z_{1e} = total equivalent impedance referred to primary

$$Z_{1e} = R_{1e} + jX_{1e} = Z_1 + Z_2' = Z_1 + \frac{Z2}{K^2}$$

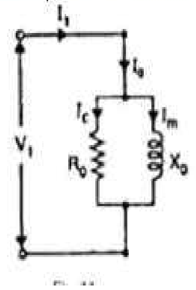
Complete Phasor Diagram of a Transformer (for Inductive Load or Lagging pf)

We now restrict ourselves to the more commonly occurring load i.e. inductive along with resistance, which has a lagging power factor. For drawing this diagram, we must remember that

$$\overline{V_2} = \overline{E_2} - \overline{I_2}\,(R_2 + j\,X_{l2})$$
$$\overline{V_1} = -\,\overline{E_1} + \overline{I_2}\,(R_1 + j\,X_{l1})$$



No load equivalent circuit



$I_m = I_o \sin\phi_o$ = magnetizing component
$I_c = Io \cos\phi_o$ = Active component

$R_o = \frac{V1}{Ic}$, $X_o = \frac{V1}{Im}$

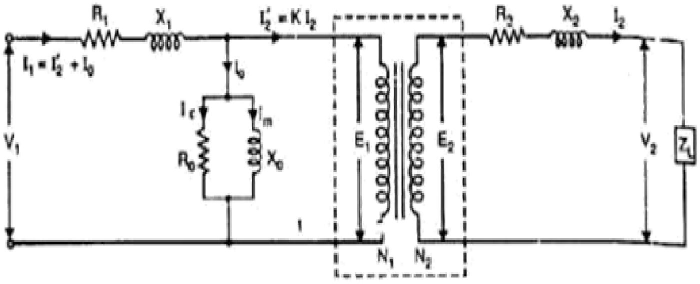
i) Im produces the flux and is assumed to flow through reactance Xo called no load reactance while Ic is active component representing core losses hence is assumed to flow through the resistance R0

ii) Equivalent resistance is shown in fig.2.12.

iii) When the load is connected to the transformer then secondary current I2 flows causes voltage drop across R2 and X2. Due to I2, primary draws an additional current.

I1 is the phasor addition of Io and I2'. This I1 causes the voltage drop across primary resistance R1 and reactance X1.

Equivalent Circuit of Transformer



Exact equivalent circuit referred to primary

To simplified the circuit the winding is not taken in equivalent circuit while transfer to one side.

Exact equivalent circuit referred to primary

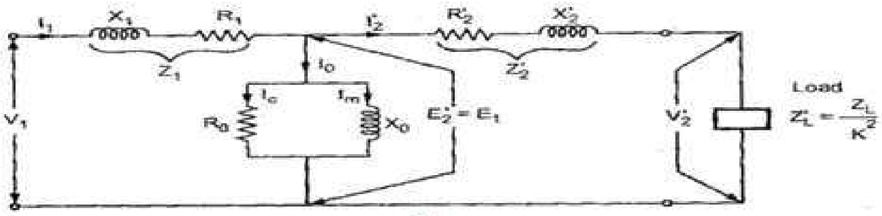
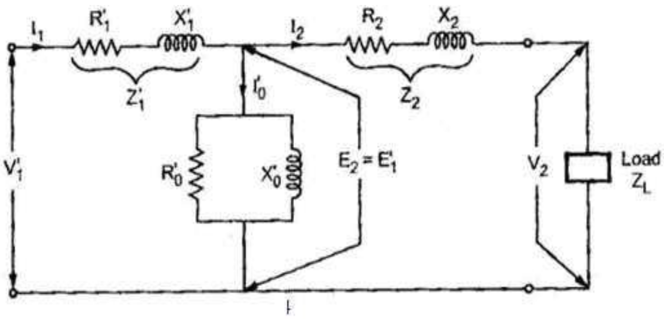


Fig:

Transferring secondary parameter to primary -

$$R_2' = \frac{R2}{K^2},\ X_2' = \frac{X2}{K^2},\ Z_2' = \frac{Z2}{K^2},\ E_2' = \frac{E2}{K'},\ I_2' = KI_2,\ K = \frac{N2}{N1}$$



V

Now as long as no load branch i.e. exciting branch is in between Z1 and Z2', the impedances cannot be combined. So further simplification of the circuit can be done. Such circuit is called approximate equivalent circuit.

$$R_1' = R_1K^2,\ X_1' = K^2X_1\ ,\ E_1' = KE_1$$

$$Z_1' = K^2Z1\ ,\ I_1' = \frac{I1}{K'},\ I_o = \frac{Io}{K}$$

Approximate Equivalent Circuit

i) To get approximate equivalent circuit, shift the no load branch containing Ro and Xo to the left of R1 and X1.

ii) By doing this we are creating an error that the drop across R1 and X1 to Io is neglected due to this circuit because simpler.

iii) This equivalent circuit is called approximate equivalent circuit Fig: 2.15 & Fig: 2.16.

In this circuit new R1 and R2' can be combined to get equivalent circuit referred to primary R1e,similarly

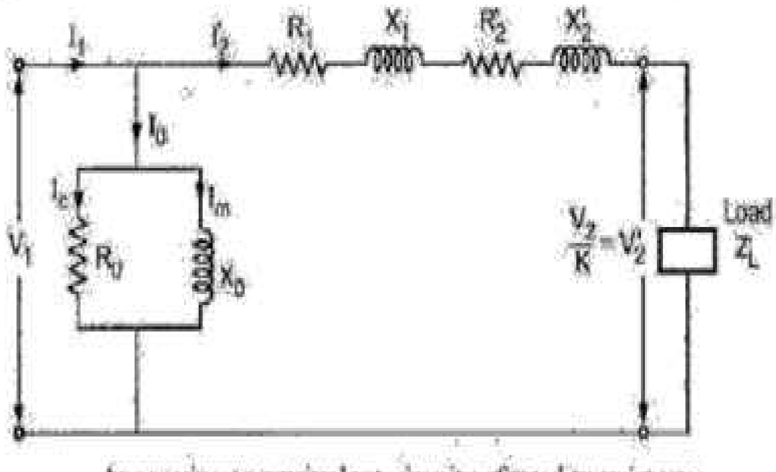
X1 and X2' can be combined to get X1e.

$$R_{1e} = R_1 + R_2' = R_1 + \frac{R2}{K^2}$$

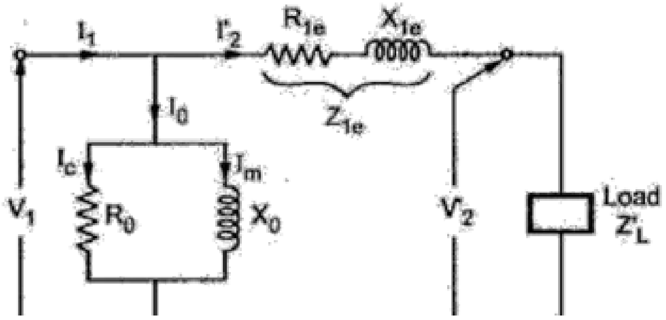
$$X_{1e} = X_1 + X_2' = X_1 + \frac{X2}{K^2}$$

$$Z_{1e} = R_{1e} + jX_1E, \qquad R_o = \frac{V1}{Ic}, \quad \text{and } X_o = \frac{V1}{Im}$$

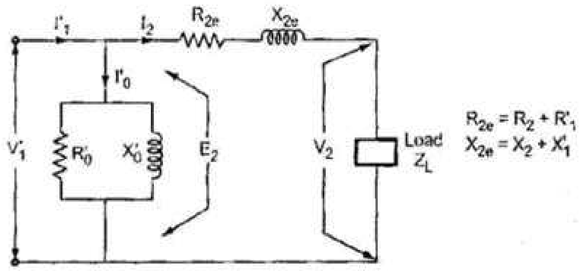
$$I_c = I_o\cos\phi_o \quad \text{and } I_m = I_o\sin\phi_o$$



Approximate equivalent circuit referred to primary



Approximate Voltage Drop in a Transformer



$$\vec{E_2} = \vec{I_2 R_{2e}} + \vec{I_2 X_{2e}} + \vec{V_2} = \vec{V_2} + \vec{I_2}(R_{2e} + jX_{2e}), \quad \vec{E_2} = \vec{V_2} + \vec{I_2 Z_{2e}}$$

Primary parameter is referred to secondary there are no voltage drop in primary. When there is no load,

I2 = 0 and we get no load terminal voltage drop in V2o = E2 = no load terminal voltage

V2 = terminal voltage on load

For Lagging P.F.

i) The current I2 lags V2 by angle ϕ2

ii) Take V2 as reference

iii) I2R2e is in phase with I2 while I2 X2e leads I2 by 90˚

iv) Draw the circle with O as centre and OC as radius cutting extended OA at M. as OA = V2 and now OM = E2.

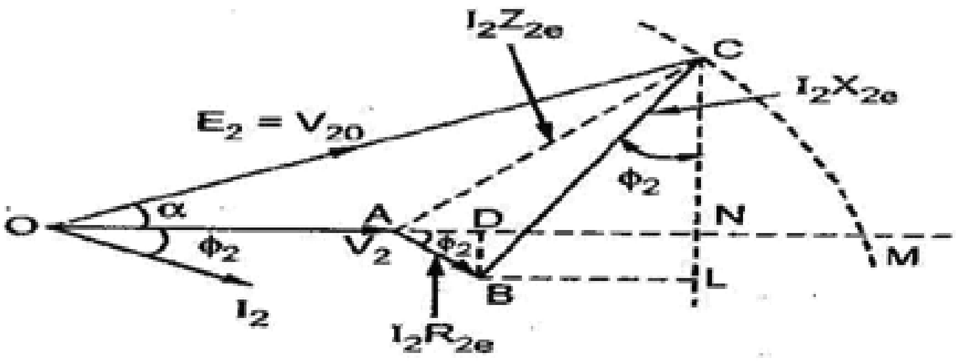
v) The total voltage drop is AM = I2Z2e.

vi) The angle α is practically very small and in practice M&N are very close to each other. Due to this the approximate voltage drop is equal to AN instead of AM AN – approximate voltage drop To find AN by adding AD& DN AD = AB cosϕ

= I2R2e cosϕ DN = BL sinϕ = I2X2e sinϕ

AN = AD + DN = I2R2e cosϕ2 + I2X2e sinϕ2 Assuming: ϕ2 = ϕ1 = ϕ

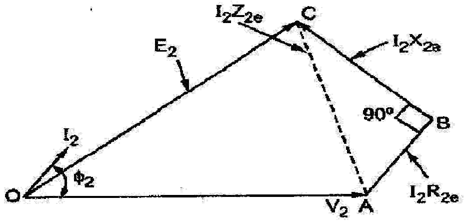
Approximate voltage drop = I2R2e cosϕ+I2X2e sinϕ (referred to secondary) Similarly: Approximate voltage drop = I1R1e cosϕ+I1X1e sinϕ (referred to primary)



For Leading P.F Loading

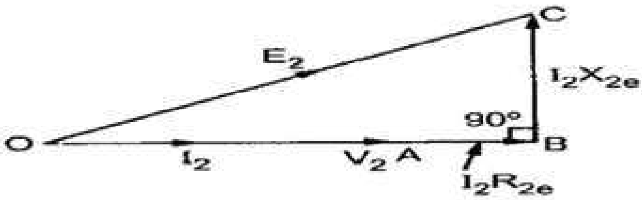
I2 leads V2 by angle ϕ2

Approximate voltage drop = I2R2e cosϕ - I2X2e sinϕ (referred to secondary) Similarly: Approximate voltage drop = I1R1e cosϕ - I1X1e sinϕ (referred to primary)



For Unity P.F. Loading

Approximate voltage drop = I2R2e (referred to secondary) Similarly: Approximate voltage drop = I1R1e (referred to primary



Cosϕ = 1
Sinϕ = 0

Approximate voltage drop = E2 – V2
= I2R2e cosϕ ± I2X2e sinϕ (referred to secondary)
= I1R1e cosϕ ± I1X1e sinϕ (referred to primary)

Losses in a Transformer

The power losses in a transformer are of two types, namely;

1.Core or Iron losses

2.Copper losses

These losses appear in the form of heat and produce (i) an increase in Temperature and (ii) a drop in efficiency.

Core or Iron losses (Pi)

These consist of hysteresis and eddy current losses and occur in the transformer core due to the alternating flux. These can be determined by open-circuit test.

Hysteresis loss = kh f Bm1.6 watts /m3

Kh – hysteresis constant depend on material f - Frequency

Bm – maximum flux density

Eddy current loss = Ke f2 Bm2t2 watts /m3

Ke – eddy current constant t - Thickness of the core

Both hysteresis and eddy current losses depend upon

(i)Maximum flux density Bm in the core

(ii)Supply frequency f. Since transformers are connected to constant-frequency, constant voltage supply, both f and Bm are constant. Hence, core or iron losses are practically the same at all lods.

Iron or Core losses, Pi = Hysteresis loss + Eddy current loss = Constant losses (Pi)

The hysteresis loss can be minimized by using steel of high silicon content .Whereas eddy current loss can be reduced by using core of thin laminations.

Copper losses (Pcu)

These losses occur in both the primary and secondary windings due to their ohmic resistance. These can be determined by short-circuit test. The copper loss depends on the magnitude of the current flowing through the windings.

$$\text{Total copper loss} = I_1^2R_1 + I_2^2R_2 = I_1^2(R_1 + R_2') = I_2^2(R_2 + R_1')$$

$$\boxed{\text{Total loss} = \text{iron loss} + \text{copper loss} = Pi + Pcu}$$

Efficiency of a Transformer

Like any other electrical machine, the efficiency of a transformer is defined as the ratio of output power (in watts or kW) to input power (watts or kW) i.e.

Power output = power input – Total losses Power input = power output + Total losses

= power output + Pi + Pcu

$$\text{Efficiency} = \frac{power\ output}{power\ input}$$

$$\text{Efficiency} = \frac{power\ output}{power\ input + Pi + Pcu}$$

Power output = $V_2 I_2 \cos\phi$, $\cos\phi$ = load power factor

Transformer supplies full load of current I_2 and with terminal voltage V_2

Pcu = copper losses on full load = $I_2^2 R_{2e}$

This is full load efficiency and I2 = full load current. We can now find the full-load efficiency of the transformer at any p.f. without actually loading the transformer.

$$\text{Full load Efficiency} = \frac{(\text{Full load VA rating}) X \cos\phi}{(\text{Full load VA rating}) X \cos\phi + Pi + I2^2 R2e}$$

Also for any load equal to n x full-load,

Corresponding total losses = $P_i + n^2 P_{cu}$

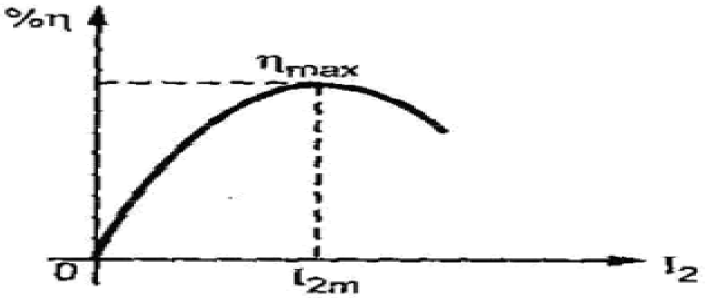
$$n = \text{fractional by which load is less than full load} = \frac{actual\ load}{full\ load}$$

$$n = \frac{half\ load}{fullload} = \frac{(\frac{1}{2})}{1} = 0.5$$

$$\text{Corresponding (n) \% Efficiency} = \frac{n(\text{VA rating}) X \cos\phi}{n(\text{VA rating}) X \cos\phi + Pi + n^2 Pcu} X\ 100$$

Condition for Maximum Efficiency

Voltage and frequency supply to the transformer is constant the efficiency varies with the load. As load increases, the efficiency increases. At a certain load current, it loaded further the efficiency start decreases as shown in



The load current at which the efficiency attains maximum value is denoted as I2m a n d maximum efficiency is denoted as ηmax, now we find -

(a)condition for maximum efficiency

(b)load current at which ηmax occurs

(c)KVA supplied at maximum efficiency Considering primary side,

Load Output $=v_1 I_1 \cos\phi_1$

Copper loss = $I^2 R$

Iron loss = hysteresis + eddy current loss = Pi

$$\text{Efficiency} = \frac{V1I1\cos\phi1 - \text{losses}}{V1I1\cos\phi1} = \frac{V1I1\cos\phi1 - I1^2R1e + Pi}{V1I1\cos\phi1}$$

$$= 1 - \frac{I1R1e}{V1I21\cos\phi1} = \frac{Pi}{V1I1\cos\phi1}$$

Differentiating both sides with respect to I2, we get

$$\frac{d\eta}{dI2} = 0 - \frac{R1e}{V1\cos\phi1} = \frac{Pi}{V1I1^2\cos\phi1}$$

For η to be maximum, $\frac{d\eta}{dI2} = 0$. Hence, the above equation becomes

$$\frac{R1e}{V1\cos\phi1} = \frac{Pi}{V1I1^2\cos\phi1} \text{ or } Pi = I_1^2 R_{1e}$$

Pcu loss = Pi iron loss

The output current which will make Pcu loss equal to the iron loss. By proper design, it is possible to make the maximum efficiency occur at any desired load.

Load current I2m at maximum efficiency KVA Supplied at Maximum Efficiency

For constant V2 the KVA supplied is the function of load current.

For η_{max} $I_2^2 R_{2e} = P_i$ but $I_2 = I_{2m}$

$$I^2_{2m} R_{2e} = Pi \qquad I_{2m} = \sqrt{\frac{Pi}{R2e}}$$

This is the load current at η_{max}.

(I_2)F.L = full load current

$$\frac{I2m}{(I2)F.L} = \frac{1}{(I2)F.L}\sqrt{\frac{Pi}{R2e}}$$

$$\frac{I2m}{(I2)F.L} = \sqrt{\frac{Pi}{[(I2)F.L]^2 R2e}} = \sqrt{\frac{Pi}{[Pcu]F.L}}$$

$$\boxed{I_{2m} = (I_2)\,F.L.\sqrt{\frac{Pi}{[Pcu]F.L}}}$$

KVA Supplied at Maximum Efficiency
For constant V2 the KVA supplied is the function of load current.

$$\text{KVA at } \eta_{max} = I_{2m} V_2 = V_2(I_2)\text{F.L. X } \sqrt{\frac{Pi}{[Pcu]\text{F.L}}}$$

$$\boxed{\text{KVA at } \eta_{max} = (\text{KVA rating}) \text{ X } \sqrt{\frac{Pi}{[Pcu]\text{F.L}}}}$$

Substituting condition for η_{max} in the expression of efficiency, we can write expression for η_{max} as ,

as Pcu = Pi

$$\% \, \eta_{max} = \frac{V2 \, I2m \cos\phi}{V2 \, I2m\cos\phi + 2Pi} \text{ X } 100$$

TESTING OF TRANSFORMERS

Testing of Transformer
The testing of transformer means to determine efficiency and regulation of a transformer at any load and at any power factor condition.
There are two methods

i)Direct loading test

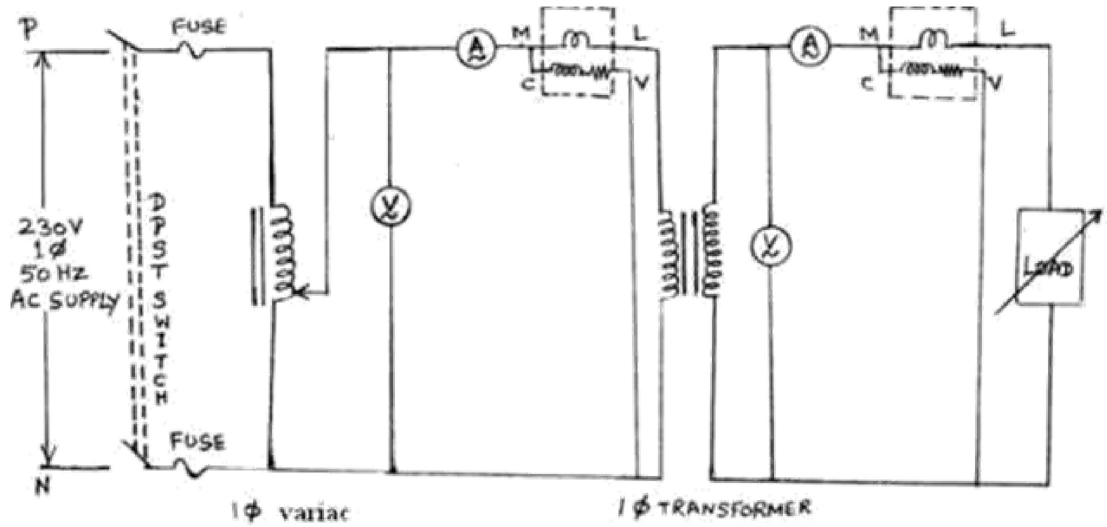
ii)Indirect loading test

a.Open circuit test

b.Short circuit test

i)Load test on transformer

This method is also called as direct loading test on transformer because the load is directly connected to the transformer. We required various meters to measure the input and output reading while change the load from zero to full load. Fig. 2.22 shows the connection of transformer for direct load test. The primary is connected through the variac to change the input voltage as we required. Connect the meters as shown in the figure below.



The load is varied from no load to full load in desired steps. All the time, keep primary voltage V1 constant at its rated value with help of variac and tabulated the reading. The first reading is to be noted on no load for which I2 = 0 A and W2 = 0W.

Calculation

From the observed reading

W1 = input power to the transformer W2 = output power delivered to the load

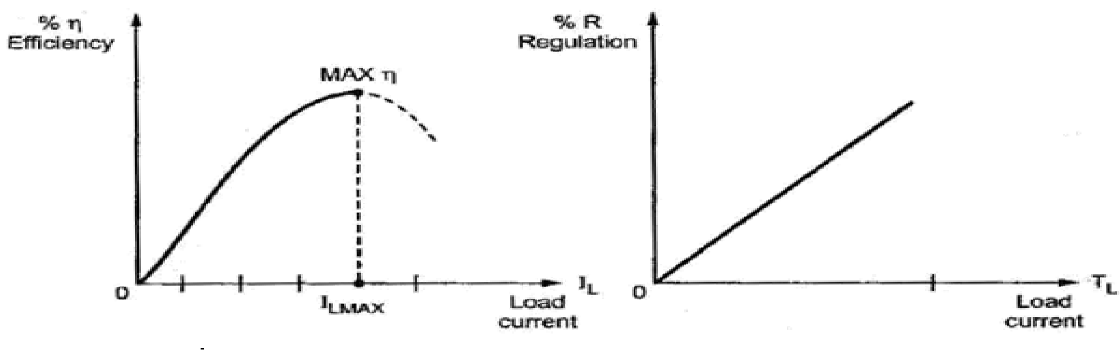
$$\% \eta = \frac{W2}{W1} X\ 100$$

The first reading is no load so V2 = E_2

The regulation can be obtained as

%R= E2-V2/E2

The graph of % η and % R on each load against load current IL is plotted as shown in fig.



Advantages:

1) This test enables us to determine the efficiency of the transformer accurately at any load.

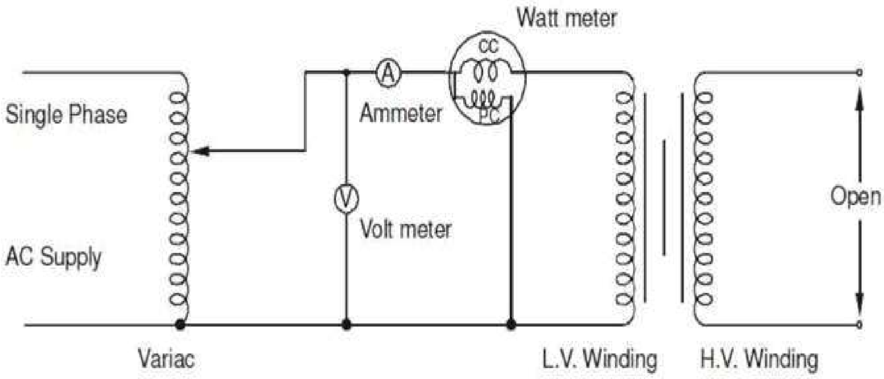
2) The results are accurate as load is directly used.

Disadvantages:
1) There are large power losses during the test.
2) Load not avail in lab while test conduct for large transformer.

ii)a. Open-Circuit or No-Load Test

This test is conducted to determine the iron losses (or core losses) and parameters R0 and X0 of the transformer. In this test, the rated voltage is applied to the primary (usually low-voltagewinding) while the secondary is left open circuited. The applied primary voltage V1 is measured by the voltmeter, the no load current I0 by ammeter and no-load input power W0 by wattmeter as shown in Fig..

As the normal rated voltage is applied to the primary, therefore, normal iron losses will occur in the transformer core. Hence wattmeter will record the iron losses and small copper loss in the primary. Since no-load current I0 is very small (usually 2-10 % of rated current). Cu losses in the primary under no-load condition are negligible as compared with iron losses.
Hence, wattmeter reading practically gives the iron losses in the transformer. It is reminded that iron losses are the same at all loads.



Iron losses, P_i = Wattmeter reading = W_0
No load current = Ammeter reading = I_0
Applied voltage = Voltmeter reading = V_1

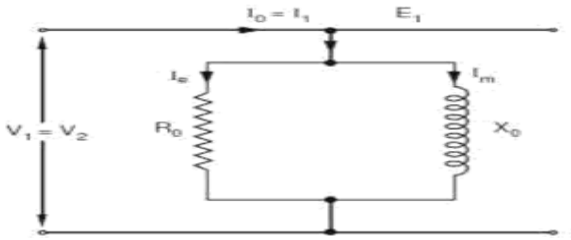
Input power, $W_0 = V_1 I_0 \cos \phi_0$

No - load p.f., $\cos\phi = \frac{Wo}{Vo\,Io}$ = no load power factor

$I_m = I_o \sin \phi_o$ = magnetizing component
$I_c = I_o \cos \phi_o$ = Active component

$R_o = \frac{Vo}{Ic}\ \Omega$, $X_o = \frac{Vo}{Im}\ \Omega$

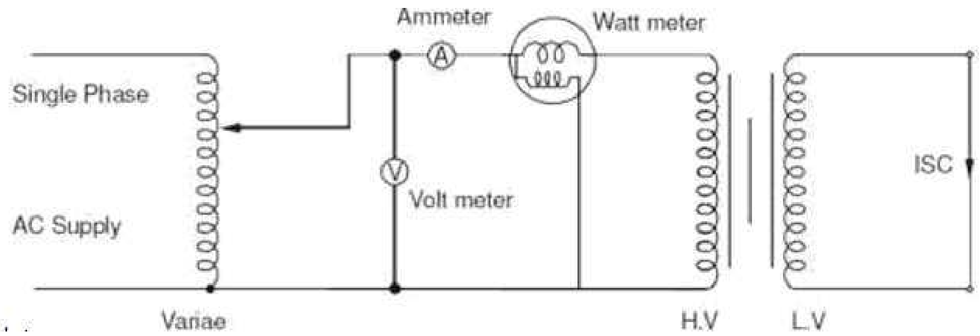
Under no load conditions the PF is very low (near to 0) in lagging region. By using the above data we can draw the equivalent parameter shown in Figure



Thus open-circuit test enables us to determine iron losses and parameters R0 and X0 of the transformer

ii)b. Short-Circuit or Impedance Test

This test is conducted to determine R1e (or R2e), X1e (or X2e) and full-load copper losses of the transformer. In this test, the secondary (usually low-voltage winding) is short-circuited by a thick conductor and variable low voltage is applied to the primary as shown in Fig.2.25. The low input voltage is gradually raised till at voltage VSC, full-load current I1 flows in the primary. Then I2 in the secondary also has full-load value since I1/I2 = N2/N1. Under such conditions, the copper loss in the windings is the same as that on full load. There is no output from the transformer under short-circuit conditions. Therefore, input power is all loss and this loss is almost entirely copper loss. It is because iron loss in the core is negligibly small since the voltage VSC is very small. Hence, the wattmeter will practically register the full load copper losses in the transformer windings.

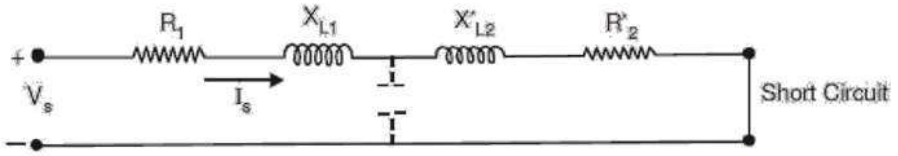


$P_{cu} = I_1^2 R_1 + I_1^2 R_2' = I_1^2 R_{1e}$, $R_{1e} = \frac{Pcu}{I1^2}$

Where R_{1e} is the total resistance of transformer referred to primary.

Total impedance referred to primary, $Z1e = \sqrt{Z1e^2 - R1e^2}$,

short – circuit P.F. $\cos\Phi = \frac{Pcu}{Vsc\ I1}$ This short-circuit lest gives full-load Cu loss, R1e and X1e.



$$\text{equivalent resistance } R_{eq} = \frac{W_s}{I_s^2} = R_1 + R_2'$$

$$\text{and equivalent impedance } Z_{eq} = \frac{V_s}{I_s}$$

So we calculate equivalent reactance

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} = X_{L1} + X'_{L2}$$

These R_{eq} and X_{eq} are equivalent resistance and reactance of both windings referred in HV side. These are know as equivalent circuit resistance and reactance.

Voltage Regulation of Transformer

Under no load conditions, the voltage at the secondary terminals is E2 and

$$E_2 = V_1 \cdot \frac{N_2}{N_1}$$

(This approximation neglects the drop R1 and Xl1 due to small no load current). As load is applied to the transformer, the load current or the secondary current increases. Correspondingly, the primary current I1 also increases. Due to these currents, there is a voltage drop in the primary and secondary leakage reactances, and as a consequence the voltage across the output terminals or the load terminals changes. In quantitative terms this change in terminal voltage is called Voltage Regulation.

Voltage regulation of a transformer is defined as the drop in the magnitude of load voltage (or secondary terminal voltage) when load current changes from zero to full load value. This is expressed as a fraction of secondary rated voltage.

$$\text{Regulation} = \frac{\text{Secondary terminal voltage at no load} - \text{Secondary terminal voltage at any load}}{\text{Secondary rated voltage}}$$

The secondary rated voltage of a transformer is equal to the secondary terminal voltage at no load (i.e. E2), this is as per IS.

Voltage regulation is generally expressed as a percentage.

$$\text{Percent voltage regulation (\% VR)} = \frac{E_2 - V_2}{E_2} \times 100.$$

Note that E2, V2 are magnitudes, and not phasor or complex quantities. Also note that voltage regulation depends not only on load current, but also on its power factor. Using approximate equivalent circuit referred to primary or secondary, we can obtain the voltage regulation. From approximate equivalent circuit referred to the secondary side and phasor diagram for the circuit.

$$E_2 = V_2 + I_2\, r_{eq} \cos\phi_2 \pm I_2\, x_{eq} \sin\phi_2$$

where $r_{eq} = r_2 + r_1^1$ (referred to secondary) $x_e = x_2 + x_1^1$ (+ sign applies lagging power factor load and – sign applies to leading pf load).

So
$$\frac{E_2 - V_2}{E_2} = \frac{I_2\, r_{eq} \cos\phi_2 \pm I_2\, x_{eq} \sin\phi_2}{E_2}$$

$$\frac{E_2 - V_2}{E_2} = \frac{I_2\, r_{eq}}{E_2} \cos\phi_2 \pm \frac{I_2\, x_{eq}}{E_2} \sin\phi_2$$

% Voltage regulation = (% resistive drop) cos ϕ_2 ± (% reactive drop) sin ϕ_2.

Ideally voltage regulation should be zero.

Induction Motor

The most common type of AC motor being used throughout the work today is the "Induction Motor". Applications of three-phase induction motors of size varying from half a kilowatt to thousands of kilowatts are numerous. They are found everywhere from a small workshop to a large manufacturing industry.
The advantages of three-phase AC induction motor are listed below:

- Simple design
- Rugged construction
- Reliable operation
- Low initial cost
- Easy operation and simple maintenance
- Simple control gear for starting and speed control
- High efficiency.

Induction motor is originated in the year 1891 with crude construction (The induction machine principle was invented by NIKOLA TESLA in 1888.). Then an improved construction with distributed stator windings and a cage rotor was built.

The slip ring rotor was developed after a decade or so. Since then a lot of improvement has taken place on the design of these two types of induction motors. Lot of research work has been carried out to improve its power factor and to achieve suitable methods of speed control.

Types and Construction of Three Phase Induction Motor

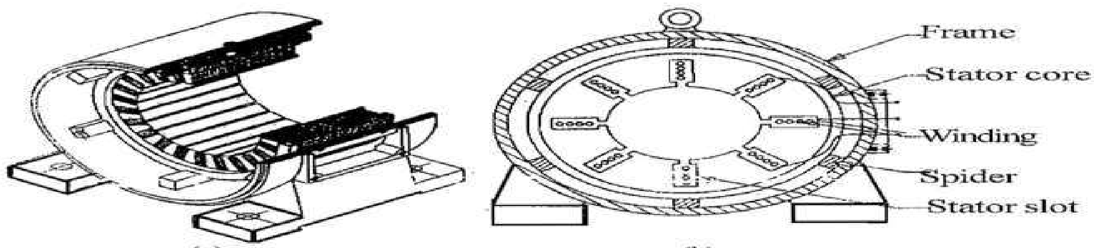
Three phase induction motors are constructed into two major types:

1. Squirrel cage Induction Motors
2. Slip ring Induction Motors

Squirrel cage Induction Motors

(a)Stator Construction

The induction motor stator resembles the stator of a revolving field, three phase alternator. The stator or the stationary part consists of three phase winding held in place in the slots of a laminated steel core which is enclosed and supported by a cast iron or a steel frame as shown in Fig: 3.1(a). The phase windings are placed 120 electrical degrees apart and may be connected in either star or delta externally, for which six leads are brought out to a terminal box mounted on the frame of the motor. When the stator is energized from a three phase voltage it will produce a rotating magnetic field in the stator core.



(a) (b)

(b) Rotor Construction

The rotor of the squirrel cage motor shown in Fig: 3.1(b) contains no windings. Instead it is a cylindrical core constructed of steel laminations with conductor bars mounted parallel to the shaft and embedded near the surface of the rotor core.

These conductor bars are short circuited by an end rings at both end of the rotor core. In large machines, these conductor bars and the end rings are made up of copper with the bars brazed or welded to the end rings shown in Fig: 3.1(b).In small machines the conductor bars and end rings are sometimes made of aluminium with the bars and rings cast in as part of the rotor core. Actually the entire construction (bars and end-rings) resembles a squirrel cage, from which the name is derived.

The rotor or rotating part is not connected electrically to the power supply but has voltage induced in it by transformer action from the stator. For this reason, the stator is sometimes called the primary and the rotor is referred to as the secondary of the motor since the motor operates on the principle of induction and as the construction of the rotor with the bars and end rings resembles a squirrel cage, the squirrel cage induction motor is used.

The rotor bars are not insulated from the rotor core because they are made of metals having less resistance than the core. The induced current will flow mainly in them. Also the rotor bars are usually not quite parallel to the rotor shaft but are mounted in a slightly skewed position. This feature tends to produce a more uniform rotor field and torque. Also it helps to reduce some of the internal magnetic noise when the motor is running.

(a)End Shields

The function of the two end shields is to support the rotor shaft. They are fitted with bearings and attached to the stator frame with the help of studs or bolts attention.

Slip ring Induction Motors

(b)Stator Construction

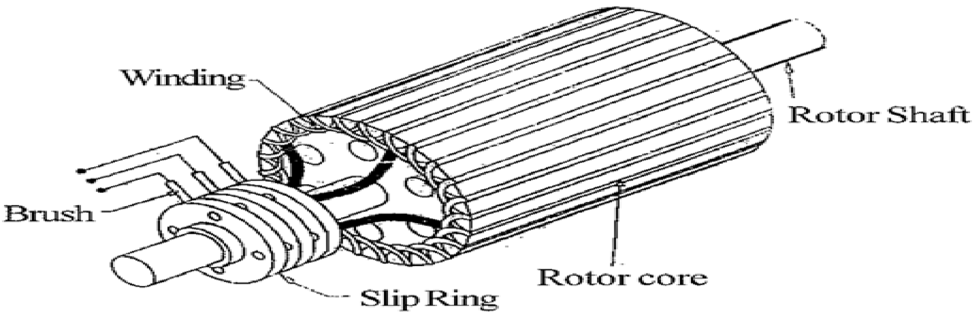
The construction of the slip ring induction motor is exactly similar to the construction of squirrel cage induction motor. There is no difference between squirrel cage and slip ring motors.

(c)Rotor Construction

The rotor of the slip ring induction motor is also cylindrical or constructed of lamination.

Squirrel cage motors have a rotor with short circuited bars whereas slip ring motors have wound rotors having "three windings" each connected in star.

The winding is made of copper wire. The terminals of the rotor windings of the slip ring motors are brought out through slip rings which are in contact with stationary brushes as shown in Fig:



THE ADVANTAGES OF THE SLIPRING MOTOR ARE

- It has susceptibility to speed control by regulating rotor resistance.
- High starting torque of 200 to 250% of full load value.
- Low starting current of the order of 250 to 350% of the full load current.

Hence slip ring motors are used where one or more of the above requirements are to be met.

Sl. No.	Property	*Squirrel cage motor*	*Slip ring motor*
1.	**Rotor Construction**	*Bars are used in rotor. Squirrel cage motor is very simple, rugged and long lasting. No slip rings and brushes*	*Winding wire is to be used.* *Wound rotor required attention.* *Slip ring and brushes are needed also need frequent maintenance.*
2.	**Starting**	*Can be started by D.O.L., star-delta, auto transformer starters*	*Rotor resistance starter is required.*
3.	**Starting torque**	*Low*	*Very high*
4.	**Starting Current**	*High*	*Low*

5.	**Speed variation**	*Not easy, but could be varied in large steps by pole changing or through smaller incremental steps through thyristors or by frequency variation.*	*Easy to vary speed.* *Speed change is possible by inserting rotor resistance using thyristors or by using frequency variation injecting emf in the rotor circuit cascading.*
6.	**Maintenance**	*Almost ZER*	*Requires frequent maintenance*
7.	**Cost**	*Low*	*High*

Principle of Operation

The operation of a 3-phase induction motor is based upon the application of Faraday Law and the Lorentz force on a conductor. The behaviour can readily be understood by means of the following example.

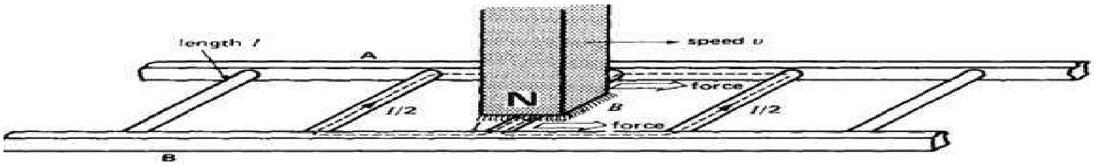
Consider a series of conductors of length l, whose extremities are short-circuited by two bars A and B (Fig. a). A permanent magnet placed above this conducting ladder, moves rapidly to the right at a speed v, so that its magnetic field B sweeps across the conductors. The following sequence of events then takes place:

1.A voltage E = Blv is induced in each conductor while it is being cut by the flux (Faraday law).

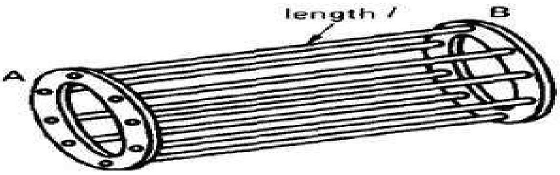
2.The induced voltage immediately produces a current I, which flows down the conductor underneath the pole face, through the end-bars, and back through the other conductors.

3.Because the current carrying conductor lies in the magnetic field of the permanent magnet, it experiences a mechanical force (Lorentz force).

4.The force always acts in a direction to drag the conductor along with the magnetic field. If the conducting ladder is free to move, it will accelerate toward the right. However, as it picks up speed, the conductors will be cut less rapidly by the moving magnet, with the result that the induced voltage E and the current I will diminish. Consequently, the force acting on the conductors wilt also decreases. If the ladder were to move at the same speed as the magnetic field, the induced voltage E, the current I, and the force dragging the ladder along would all become zero.



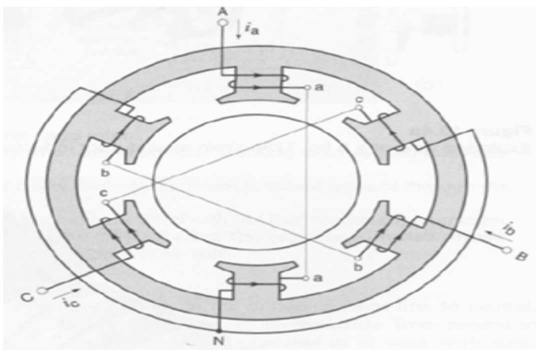
(a)



In an induction motor the ladder is closed upon itself to form a squirrel-cage (Fig.3.3b) and the moving magnet is replaced by a rotating field. The field is produced by the 3-phase currents that flow in the stator windings.

Rotating Magnetic Field and Induced Voltages

Consider a simple stator having 6 salient poles, each of which carries a coil having 5 turns (Fig.). Coils that are diametrically opposite are connected in series by means of three jumpers that respectively connect terminals a-a, b-b, and c-c. This creates three identical sets of windings AN, BN, CN, which are mechanically spaced at 120 degrees to each other. The two coils in each winding produce magneto motive forces that act in the same direction.
The three sets of windings are connected in wye, thus forming a common neutral N. Owing to the perfectly symmetrical arrangement, the line to neutral impedances are identical. In other words, as regards terminals A, B, C, the windings constitute a balanced 3-phase system.
For a two-pole machine, rotating in the air gap, the magnetic field (i.e., flux density) being sinusoidally distributed with the peak along the centre of the magnetic poles. The result is illustrated in Fig.3.5. The rotating field will induce voltages in the phase coils aa', bb', and cc'. Expressions for the induced voltages can be obtained by using Faraday laws of induction.



· Elementary stator having terminals A, B, C connected to a 3-phase source (not shown). Currents flowing from line to neutral are considered to be positive.

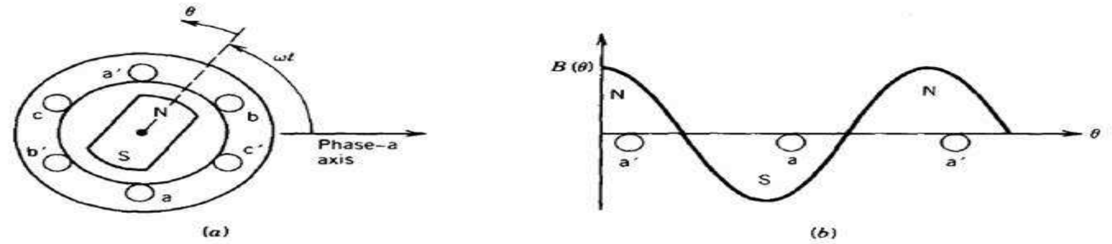


Fig: 3.5 Air gap flux density distribution.

The flux density distribution in the air gap can be expressed as:

$$B(\theta) = B_{\max} \cos\theta$$

The air gap flux per pole, ϕ_P, is:

$$\phi_p = \int_{-\pi/2}^{\pi/2} B(\theta) lr d\theta = 2B_{\max} lr$$

Where,

l is the axial length of the stator.

r is the radius of the stator at the air gap.

Let us consider that the phase coils are full-pitch coils of N turns (the coil sides of each phase are 180 electrical degrees apart as shown in Fig.3.5). It is obvious that as the rotating field moves (or the magnetic poles rotate) the flux linkage of a coil will vary. The flux linkage for coil aa' will be maximum.

($= N\phi_P$ at $\omega t = 0^o$) (Fig.3.5a) and zero at $\omega t = 90^o$. The flux linkage $\lambda_a(\omega t)$ will vary as the cosine of the angle ωt.

Hence,

$$\lambda_a(\omega t) = N\phi_p \cos \omega t$$

Therefore, the voltage induced in phase coil **aa'** is obtained from *Faraday law* as:

$$e_a = -\frac{d\lambda_a(\omega t)}{dt} = \omega N\phi_p \sin \omega t = E_{\max} \sin \omega t$$

The voltages induced in the other phase coils are also sinusoidal, but phase-shifted from each other by 120 electrical degrees. Thus,

$$e_b = E_{\max} \sin(\omega t - 120)$$

$$e_c = E_{\max} \sin(\omega t + 120).$$

the *rms* value of the induced voltage is:

$$E_{rms} = \frac{\omega N\phi_p}{\sqrt{2}} = \frac{2\pi f}{\sqrt{2}} N\phi_p = 4.44 fN\phi_p$$

Where f is the frequency in hertz. Above equation has the same form as that for the induced voltage in transformers. However, ØP represents the flux per pole of the machine.

The above equation also shows the rms voltage per phase. The N is the total number of series turns per phase with the turns forming a concentrated full-pitch winding. In an actual AC machine each phase winding is distributed in a number of slots for better use of the iron and copper and to improve the waveform. For such a distributed winding, the EMF induced in various coils placed in different slots are not in time phase, and therefore the phasor sum of the EMF is less than their numerical sum when they are connected in series for the phase winding. A reduction factor KW, called the winding factor, must therefore be applied. For most three-phase machine windings KW is about 0.85 to 0.95.
Therefore, for a distributed phase winding, the rms voltage per phase is

Erms = 4.44fNphφpKW

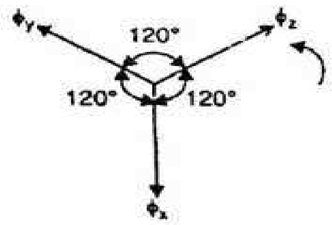
Where Nph is the number of turns in series per phase.

Alternate Analysis for Rotating Magnetic Field

When a 3-phase winding is energized from a 3-phase supply, a rotating magnetic field is produced. This field is such that its poles do no remain in a fixed position on the stator but go on shifting their positions around the stator. For this reason, it is called a rotating Held. It can be shown that magnitude of this rotating field is constant and is equal to 1.5 m where m is the maximum flux due to any phase.

To see how rotating field is produced, consider a 2-pole, 3-phase winding as shown in Fig. 3.6 (i). The three phases X, Y and Z are energized from a 3-phase source and currents in these phases are indicated as Ix, Iy and Iz [See Fig. 3.6 (ii)]. Referring to Fig. 3.6 (ii), the fluxes produced by these currents are given by:

$$\phi_x = \phi_m \sin \omega t$$
$$\phi_y = \phi_m \sin (\omega t - 120°)$$
$$\phi_z = \phi_m \sin (\omega t - 240°)$$



Here □m is the maximum flux due to any phase. Above figure shows the phasor diagram of the three fluxes. We shall now prove that this 3-phase supply produces a rotating field of constant magnitude equal to 1.5 □m.

At instant 1 [See Fig. (ii) and Fig. 3 (iii)], the current in phase X is zero and currents in phases Y and Z are equal and opposite. The currents are flowing outward in the top conductors and inward in the bottom conductors. This establishes a resultant flux towards right. The magnitude of the resultant flux is constant and is equal to 1.5 □m as proved under:

At instant 1, $\omega t = 0°$. Therefore, the three fluxes are given by;

$$\phi_x = 0; \qquad \phi_y = \phi_m \sin(-120°) = -\frac{\sqrt{3}}{2}\phi_m;$$
$$\phi_z = \phi_m \sin(-240°) = \frac{\sqrt{3}}{2}\phi_m$$

The phasor sum of $-\phi_y$ and ϕ_z is the resultant flux ϕ_r

So,
At instant 2 [Fig: 3.7 (ii)], the current is maximum (negative) in □y phase Y and 0.5 maximum (positive) in phases X and Y. The magnitude of resultant flux is 1.5 □m as proved under:
At instant 2, □t = 30°. Therefore, the three fluxes are given by;

$$\text{Resultant flux, } \phi_r = 2 \times \frac{\sqrt{3}}{2}\phi_m \cos\frac{60^\circ}{2} = 2 \times \frac{\sqrt{3}}{2}\phi_m \times \frac{\sqrt{3}}{2} = 1.5\ \phi_m$$

$$\phi_x = \phi_m \sin 30^\circ = \frac{\phi_m}{2}$$

$$\phi_y = \phi_m \sin(-90^\circ) = -\phi_m$$

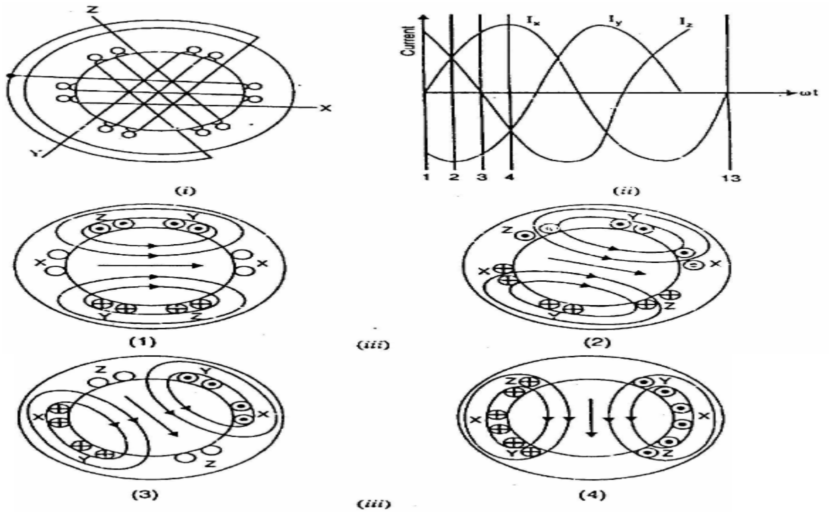
$$\phi_z = \phi_m \sin(-210^\circ) = \frac{\phi_m}{2}$$

The phasor sum of ϕ_x, $-\phi_y$ and ϕ_z is the resultant flux ϕ_r

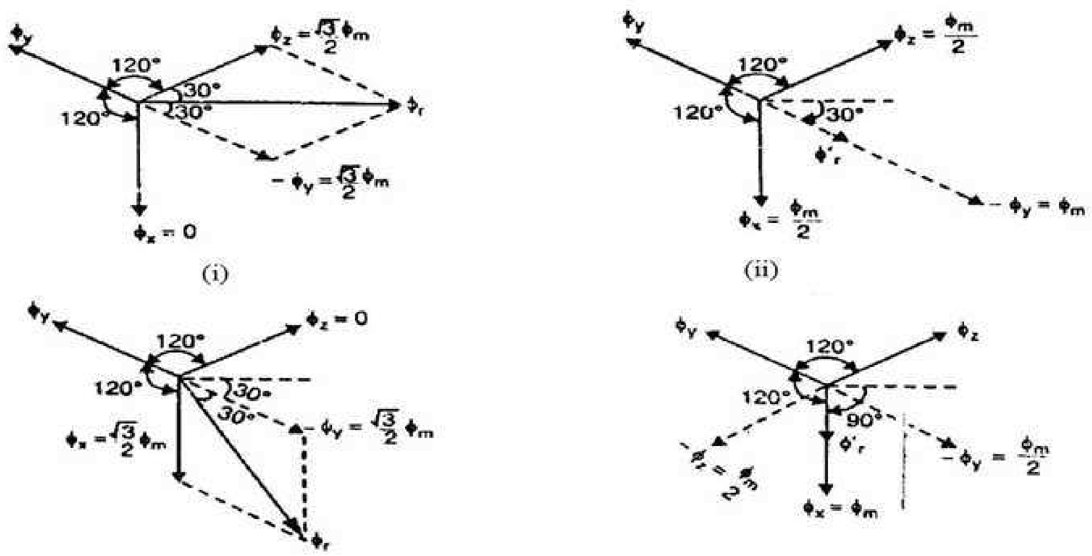
Phasor sum of ϕ_x and ϕ_z, $\phi'_r = 2 \times \frac{\phi_m}{2} \cos\frac{120^\circ}{2} = \frac{\phi_m}{2}$

Phasor sum of ϕ'_r and $-\phi_y$, $\phi_r = \frac{\phi_m}{2} + \phi_m = 1.5\ \phi_m$

Note that resultant flux is displaced 30° clockwise from position 1.



At instant 3[Fig: 3.7 (iii)], current in phase Z is zero and the currents in phases X and Y are equal and opposite (currents in phases X and Y arc 0.866 × max. value). The magnitude of resultant flux is $1.5\ \phi_m$ as proved under:



At instant 3, ωt = 60°. Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 60° = \frac{\sqrt{3}}{2}\phi_m;$$

$$\phi_y = \phi_m \sin(-60°) = -\frac{\sqrt{3}}{2}\phi_m;$$

$$\phi_z = \phi_m \sin(-180°) = 0$$

The resultant flux ϕ_r is the phasor sum of ϕ_x and $-\phi_y$ $(\because \ \phi_z = 0)$.

$$\phi_r = 2 \times \frac{\sqrt{3}}{2}\phi_m \cos\frac{60°}{2} = 1.5\ \phi_m$$

Note that resultant flux is displaced 60° clockwise from position 1.

At instant 4 [Fig: 3.7 (iv)], the current in phase X is maximum (positive) and the currents in phases V and Z are equal and negative (currents in phases V and Z are 0.5 □ max. value). This establishes a resultant flux downward as shown under:

At instant 4, $\omega t = 90°$. Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 90° = \phi_m$$

$$\phi_y = \phi_m \sin(-30°) = -\frac{\phi_m}{2}$$

$$\phi_z = \phi_m \sin(-150°) = -\frac{\phi_m}{2}$$

The phasor sum of ϕ_x, $-\phi_y$ and $-\phi_z$ is the resultant flux ϕ_r

Phasor sum of $-\phi_z$ and $-\phi_y$, $\phi'_r = 2 \times \frac{\phi_m}{2} \cos\frac{120°}{2} = \frac{\phi_m}{2}$

Phasor sum of ϕ'_r and ϕ_x, $\phi_r = \frac{\phi_m}{2} + \phi_m = 1.5\,\phi_m$

Note that the resultant flux is downward i.e., it is displaced 90° clockwise from position 1.

It follows from the above discussion that a 3-phase supply produces a rotating field of constant value (= 1.5ϕm, where m is the maximum flux due to any phase).

Speed of rotating magnetic field

The speed at which the rotating magnetic field revolves is called the synchronous speed (Ns). Referring to Fig. 3.6 (ii), the time instant 4 represents the completion of one-quarter cycle of alternating current Ix from the time instant 1. During this one quarter cycle, the field has rotated through 90°. At a time instant represented by 13 [Fig. 3.6 (ii)] or one complete cycle of current Ix from the origin, the field has completed one revolution. Therefore, for a 2-pole stator winding, the field makes one revolution in one cycle of current. In a 4-pole stator winding, it can be shown that the rotating field makes one revolution in two cycles of current. In general, fur P poles, the rotating field makes one revolution in P/2 cycles of current.

$$\therefore \quad \text{Cycles of current} = \frac{P}{2} \times \text{revolutions of field}$$

or $$\text{Cycles of current per second} = \frac{P}{2} \times \text{revolutions of field per second}$$

Since revolutions per second is equal to the revolutions per minute (N_s) divided by 60 and the number of cycles per second is the frequency f,

$$\therefore \quad f = \frac{P}{2} \times \frac{N_s}{60} = \frac{N_s P}{120}$$

or $$N_s = \frac{120\,f}{P}$$

The speed of the rotating magnetic field is the same as the speed of the alternator that is supplying power to the motor if the two have the same number of poles. Hence the magnetic flux is said to rotate at synchronous speed.

Direction of rotating magnetic field

The phase sequence of the three-phase voltage applied to the stator winding in Fig. 3.6 (ii) is X-Y-Z. If this sequence is changed to X-Z-Y, it is observed that direction of rotation of the field is reversed i.e., the field rotates counter clockwise rather than clockwise. However, the number of poles and the speed at which the magnetic field rotates remain unchanged. Thus it is necessary only to change the phase sequence in order to change the direction of rotation of
the magnetic field. For a three-phase supply, this can be done by interchanging any two of the three lines. As we shall see, the rotor in a 3-phase induction motor runs in the same direction as the rotating magnetic field. Therefore, the direction of rotation of a 3-phase induction motor can be reversed by interchanging any two of the three motor supply lines.

Slip

We have seen above that rotor rapidly accelerates in the direction of rotating field. In practice, the rotor can never reach the speed of stator flux. If it did, there would be no relative speed between the stator field and rotor conductors, no induced rotor currents and, therefore, no torque to drive the rotor. The friction and windage would immediately cause the rotor to slow down. Hence, the rotor speed (N) is always less than the suitor field speed (Ns). This difference in speed depends upon load on the motor. The difference between the synchronous speed Ns of the rotating stator field and the actual rotor speed N is called slip. It is usually expressed as a percentage of synchronous speed i.e.

$$\% \text{ age slip, } s = \frac{N_s - N}{N_s} \times 100$$

(i) The quantity $N_s - N$ is sometimes called slip speed.
(ii) When the rotor is stationary (i.e., N = 0), slip, s = 1 or 100 %.
(iii) In an induction motor, the change in slip from no-load to full-load is hardly 0.1% to 3% so that it is essentially a constant-speed motor.

Rotor Current Frequency

The frequency of a voltage or current induced due to the relative speed between a vending and a magnetic field is given by the general formula;

$$\text{Frequency} = \frac{NP}{120}$$

where N = Relative speed between magnetic field and the winding
P = Number of poles

For a rotor speed N, the relative speed between the rotating flux and the rotor is $N_s - N$. Consequently, the rotor current frequency f' is given by;

$$f' = \frac{(N_s - N)P}{120}$$

$$= \frac{sN_sP}{120} \qquad \left(\because\ s = \frac{N_s - N}{N_s}\right)$$

$$= sf \qquad \left(\because\ f = \frac{N_sP}{120}\right)$$

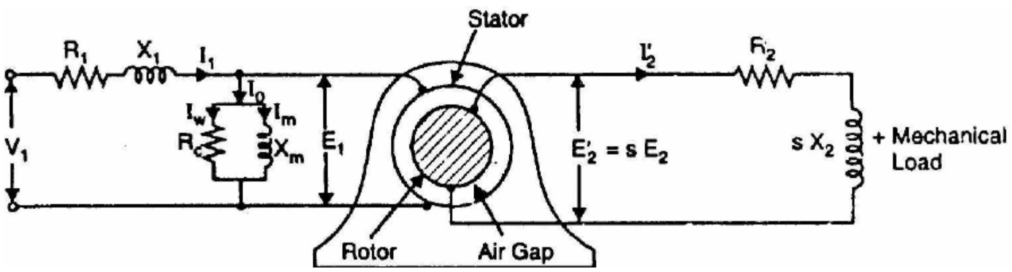
i.e., Rotor current frequency = Fractional slip x Supply frequency

(i) When the rotor is at standstill or stationary (i.e., s = 1), the frequency of rotor current is the same as that of supply frequency ($f' = sf = 1 \times f = f$).

(ii) As the rotor picks up speed, the relative speed between the rotating flux and the rotor decreases. Consequently, the slip s and hence rotor current frequency decreases.

Phasor Diagram of Three Phase Induction Motor

In a 3-phase induction motor, the stator winding is connected to 3-phase supply and the rotor winding is short-circuited. The energy is transferred magnetically from the stator winding to the short-circuited, rotor winding. Therefore, an induction motor may be considered to be a transformer with a rotating secondary (short-circuited). The stator winding corresponds to transformer primary and the rotor finding corresponds to transformer secondary. In view of the similarity of the flux and voltage conditions to those in a transformer, one can expect that the equivalent circuit of an induction motor will be similar to that of a transformer. Fig. 3.8 shows the equivalent circuit per phase for an induction motor. Let discuss the stator and rotor circuits separately.



Stator circuit. In the stator, the events are very similar to those in the transformer primary. The applied voltage per phase to the stator is V1 and R1and X1 are the stator resistance and leakage

reactance per phase respectively. The applied voltage V1 produces a magnetic flux which links the stator winding (i.e., primary) as well as the rotor winding (i.e., secondary). As a result, self-induced e.m.f. E1 is induced in the stator winding and mutually induced e.m.f.
E'2(= s E2 = s K E1 where K is transformation ratio) is induced in the rotor winding. The flow of stator current I1 causes voltage drops in R1 and X1.

$$\therefore V_1 = -E_1 + I_1 (R_1 + j X_1) \text{ ...phasor sum}$$

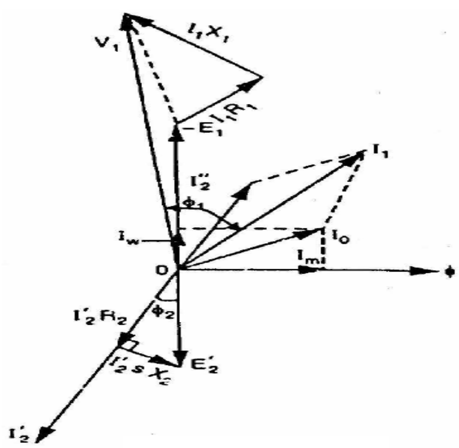
When the motor is at no-load, the stator winding draws a current I0. It has two components viz.,
(i)which supplies the no-load motor losses and
(ii) magnetizing component Im which sets up magnetic flux in the core and the air gap. The parallel combination of Rc and Xm, therefore, represents the no-load motor losses and the production of magnetic flux respectively.

$$\therefore I_0 = I_w + I_m$$

Rotor circuit. Here R2 and X2 represent the rotor resistance and standstill rotor reactance per phase respectively. At any slip s, the rotor reactance will be X2 .The induced voltage/phase in the rotor is E'2 = s E2 = s K E1. Since the rotor winding is short-circuited, the whole of e.m.f. E'2 is used up in circulating the rotor current I'2.

$$\therefore E'_2 = I'_2 (R_2 + j s X_2)$$

The rotor current I'2 is reflected as I"2 (= K I'2) in the stator. The phasor sum of I"2 and I0 gives the stator current I1.
It is important to note that input to the primary and output from the secondary of a transformer are electrical. However, in an induction motor, the inputs to the stator and rotor are electrical but the output from the rotor is mechanical. To facilitate calculations, it is desirable and necessary to replace the mechanical load by an equivalent electrical load. We then have the transformer equivalent circuit of the induction motor.



It may be noted that even though the frequencies of stator and rotor currents are different, yet the magnetic fields due to them rotate at synchronous speed Ns. The stator currents produce a magnetic flux which rotates at a speed Ns. At slip s, the speed of rotation of the rotor field relative to the rotor surface in the direction of rotation of the rotor is

$$= \frac{120\, f'}{P} = \frac{120\, s\, f}{P} = s\, N_s$$

But the rotor is revolving at a speed of N relative to the stator core. Therefore, the speed of rotor field relative to stator core

$$= sN_s + N = (N_s - N) + N = N_s$$

Thus no matter what the value of slip s, the stator and rotor magnetic fields are synchronous with each other when seen by an observer stationed in space. Consequently, the 3-phase induction motor can be regarded as being equivalent to a transformer having an air-gap separating the iron portions of the magnetic circuit carrying the primary and secondary windings. Fig. 3.9 shows the phasor diagram of induction motor.

Equivalent Circuit of Three Phase Induction Motor

Fig. (i) shows the equivalent circuit per phase of the rotor at slip s. The rotor phase current is given by;

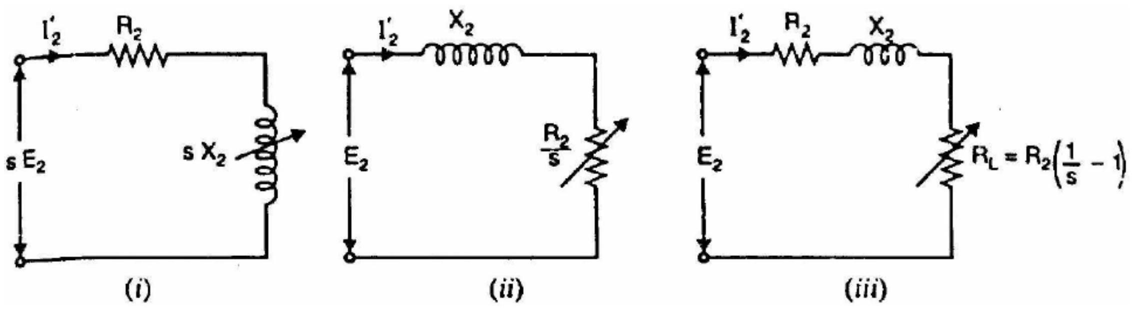
$$I'_2 = \frac{s\, E_2}{\sqrt{R_2^2 + (s\, X_2)^2}}$$

Mathematically, this value is unaltered by writing it as:

$$I'_2 = \frac{E_2}{\sqrt{(R_2 / s)^2 + (X_2)^2}}$$

As shown in Fig. 3.10 (ii), we now have a rotor circuit that has a fixed reactance X2 connected in series with a variable resistance R2/s and supplied with constant voltage E2. Note that Fig. 3.10

(ii) transfers the variable to the resistance without altering power or power factor conditions.



The quantity R_2/s is greater than R_2 since s is a fraction. Therefore, R_2/s can be divided into a fixed part R_2 and a variable part $(R_2/s - R_2)$ i.e.,

$$\frac{R_2}{s} = R_2 + R_2\left(\frac{1}{s} - 1\right)$$

(i) The first part R_2 is the rotor resistance/phase, and represents the rotor Cu loss.

(ii) The second part $R_2\left(\frac{1}{s} - 1\right)$ is a variable-resistance load. The power delivered to this load represents the total mechanical power developed in the rotor. Thus mechanical load on the induction motor can be replaced by a variable-resistance load of value $R_2\left(\frac{1}{s} - 1\right)$. This is

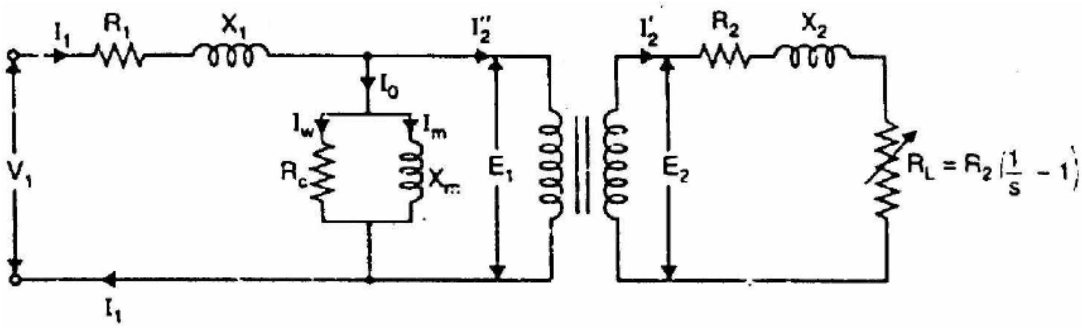
$$\therefore \quad R_L = R_2\left(\frac{1}{s} - 1\right)$$

Fig. 3.10 (iii) shows the equivalent rotor circuit along with load resistance RL.

Now Fig: 3.11 shows the equivalent circuit per phase of a 3-phase induction motor. Note that mechanical load on the motor has been replaced by an equivalent electrical resistance RL given by;

$$R_L = R_2\left(\frac{1}{s} - 1\right) \qquad \text{----- (i)}$$

The circuit shown in Fig. 3.11 is similar to the equivalent circuit of a transformer with secondary load equal to R2 given by eq. (i). The rotor e.m.f. in the equivalent circuit now depends only on the transformation ratio K (= E2/E1).



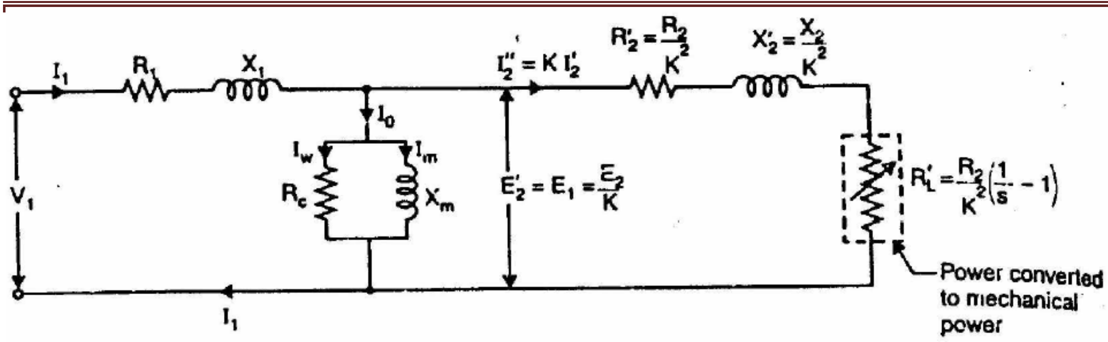
Therefore; induction motor can be represented as an equivalent transformer connected to a variable-resistance load RL given by eq. (i). The power delivered to RL represents the total mechanical power developed in the rotor. Since the equivalent circuit of Fig. 3.11 is that of a transformer, the secondary (i.e., rotor) values can be transferred to primary (i.e., stator) through the appropriate use of transformation ratio K. Recall that when shifting resistance/reactance from secondary to primary, it should be divided by K2 whereas current should be multiplied by K. The equivalent circuit of an induction motor referred to primary is shown in Fig. 3.12.

Note that the element (i.e., R'L) enclosed in the dotted box is the equivalent electrical resistance related to the mechanical load on the motor. The following points may be noted from the equivalent circuit of the induction motor.

(i) At no-load, the slip is practically zero and the load R'_L is infinite. This condition resembles that in a transformer whose secondary winding is open-circuited.

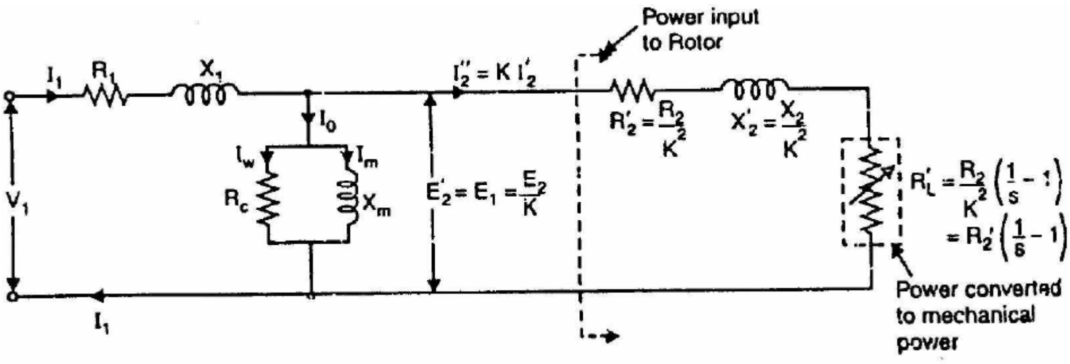
(ii) At standstill, the slip is unity and the load R'_L is zero. This condition resembles that in a transformer whose secondary winding is short-circuited.

(iii) When the motor is running under load, the value of R'_L will depend upon the value of the slip s. This condition resembles that in a transformer whose secondary is supplying variable and purely resistive load.



(iv) The equivalent electrical resistance R'_L related to mechanical load is slip or speed dependent. If the slip s increases, the load R'_L decreases and the rotor current increases and motor will develop more mechanical power. This is expected because the slip of the motor increases with the increase of load on the motor **Power and Torque Relations of Three Phase Induction Motor**

The transformer equivalent circuit of an induction motor is quite helpful in analyzing the various power relations in the motor. Fig. 3.13 shows the equivalent circuit per phase of an induction motor where all values have been referred to primary (i.e., stator).



(i) Total electrical load $= R'_2\left(\frac{1}{s}-1\right)+R'_2=\frac{R'_2}{s}$

Power input to stator $= 3V_1 I_1 \cos\phi_1$

There will be stator core loss and stator Cu loss. The remaining power will be the power transferred across the air-gap i.e., input to the rotor.

(ii) Rotor input $=\frac{3(I''_2)^2 R'_2}{s}$

Rotor Cu loss $= 3(I''_2)^2 R'_2$

Total mechanical power developed by the rotor is

P_m = Rotor input – Rotor Cu loss

$$= \frac{3(I''_2)^2 R'_2}{s} 3(I''_2)^2 R'_2 = 3(I''_2)^2 R'_2\left(\frac{1}{s}-1\right)$$

This is quite apparent from the equivalent circuit shown in

(iii) If T_g is the gross torque developed by the rotor, then,

$$P_m = \frac{2\pi N T_g}{60}$$

or $$3(I''_2)^2 R'_2 \left(\frac{1}{s} - 1\right) = \frac{2\pi N T_g}{60}$$

or $$3(I''_2)^2 R'_2 \left(\frac{1-s}{s}\right) = \frac{2\pi N T_g}{60}$$

or $$3(I''_2)^2 R'_2 \left(\frac{1-s}{s}\right) = \frac{2\pi N_s (1-s) T_g}{60} \qquad [\because N = N_s(1-s)]$$

$$\therefore \quad T_g = \frac{3(I''_2)^2 R'_2 / s}{2\pi N_s / 60} \text{ N-m}$$

or $$T_g = 9.55 \frac{3(I''_2)^2 R'_2 / s}{N_s} \text{ N-m}$$

Note that shaft torque T_{sh} will be less than T_g by the torque required to meet windage and frictional losses.

Induction Motor Torque

The mechanical power P available from any electric motor can be expressed as:

$$P = \frac{2\pi N T}{60} \text{ watts}$$

where N = speed of the motor in r.p.m.
T = torque developed in N-m

$$\therefore \quad T = \frac{60}{2\pi}\frac{P}{N} = 9.55\frac{P}{N} \text{ N-m}$$

If the gross output of the rotor of an induction motor is P_m and its speed is N r.p.m., then gross torque T developed is given by:

$$T_g = 9.55 \frac{P_m}{N} \text{ N-m}$$

Similarly, $$T_{sh} = 9.55 \frac{P_{out}}{N} \text{ N-m}$$

Note. Since windage and friction loss is small, $T_g = T_{sh}$,. This assumption hardly leads to any significant error.

Rotor Output

If T_g newton-metre is the gross torque developed and N r.p.m. is the speed of the rotor, then,

$$\text{Gross rotor output} = \frac{2\pi N T_g}{60} \quad \text{watts}$$

If there were no copper losses in the rotor, the output would equal rotor input and the rotor would run at synchronous speed Ns.

$$\therefore \quad \text{Rotor input} = \frac{2\pi N_s T_g}{60} \quad \text{watts}$$

$$\therefore \quad \text{Rotor Cu loss} = \text{Rotor input} - \text{Rotor output}$$

$$= \frac{2\pi T_g}{60}(N_s - N)$$

(i) $$\frac{\text{Rotor Cu loss}}{\text{Rotor input}} = \frac{N_s - N}{N_s} = s$$

$$\therefore \quad \text{Rotor Cu loss} = s \times \text{Rotor input}$$

(ii) Gross rotor output, P_m = Rotor input − Rotor Cu loss

= Rotor input − s × Rotor input

$$\therefore \quad P_m = \text{Rotor input}\,(1 - s)$$

(iii) $$\frac{\text{Gross rotor output}}{\text{Rotor input}} = 1 - s = \frac{N}{N_s}$$

(iv) $$\frac{\text{Rotor Cu loss}}{\text{Gross rotor output}} = \frac{s}{1 - s}$$

It is clear that if the input power to rotor is “Pr” then “s.Pr” is lost as rotor Cu loss and the remaining (1 - s) Pr is converted into mechanical power. Consequently, induction motor operating at high slip has poor efficiency.

Note.

$$\frac{\text{Gross rotor output}}{\text{Rotor input}} = 1 - s$$

If the stator losses as well as friction and windage losses arc neglected, then,

Gross rotor output = Useful output

Rotor input = Stator input

$$\therefore \quad \frac{\text{Useful output}}{\text{Stator output}} = 1 - s = \text{Efficiency}$$

Hence the approximate efficiency of an induction motor is 1 − s. Thus if the slip of an induction motor is 0.125, then its approximate efficiency is = 1 − 0.125 = 0.875 or 87.5%.

Torque Equations

The gross torque Tg developed by an induction motor is given by;

$$T_g = \frac{\text{Rotor input}}{2\pi\, N_s} \qquad \text{... } N_s \text{ is r.p.s.}$$

$$= \frac{60 \times \text{Rotor input}}{2\pi\, N_s} \qquad \text{... } N_s \text{ is r.p.s.}$$

Now $\quad \text{Rotor input} = \frac{\text{Rotor Cu loss}}{s} = \frac{3(I'_2)^2 R_2}{s} \qquad$ (i)

As shown in Sec. 8.16, under running conditions,

$$I'_2 = \frac{s\,E_2}{\sqrt{R_2^2 + (s\,X_2)^2}} = \frac{s\,K\,E_1}{\sqrt{R_2^2 + (s\,X_2)^2}}$$

where $\quad K = \text{Transformation ratio} = \frac{\text{Rotor turns/phase}}{\text{Stator turns/phase}}$

$$\therefore \quad \text{Rotor input} = 3 \times \frac{s^2\,E_2^2\,R_2}{R_2^2 + (s\,X_2)^2} \times \frac{1}{s} = \frac{3\,s\,E_2^2\,R_2}{R_2^2 + (s\,X_2)^2}$$

(Putting me value of I'_2 in eq.(i))

Also $\quad \text{Rotor input} = 3 \times \frac{s^2\,K^2\,E_1^2\,R_2}{R_2^2 + (s\,X_2)^2} \times \frac{1}{s} = \frac{3\,s\,K^2\,E_1^2\,R_2}{R_2^2 + (s\,X_2)^2}$

(Putting me value of I'_2 in eq.(i))

$$\therefore \quad T_g = \frac{\text{Rotor input}}{2\pi\, N_s} = \frac{3}{2\pi\, N_s} \times \frac{s\,E_2^2\,R_2}{R_2^2 + (s\,X_2)^2} \qquad \text{...in terms of } E_2$$

$$= \frac{3}{2\pi\, N_s} \times \frac{s\,K^2\,E_1^2\,R_2}{R_2^2 + (s\,X_2)^2} \qquad \text{...in terms of } E_1$$

Note that in the above expressions of T_g, the values E_1, E_2, R_2 and X_2 represent the phase values.

Rotor Torque

The torque T developed by the rotor is directly proportional to:
(i) rotor current
(ii) rotor e.m.f.
(iii) power factor of the rotor circuit

$$\therefore \quad T \propto E_2 I_2 \cos\phi_2$$

or $$T = K E_2 I_2 \cos\phi_2$$

where I_2 = rotor current at standstill
E_2 = rotor e.m.f. at standstill
$\cos\phi_2$ = rotor p.f. at standstill

Note. The values of rotor e.m.f., rotor current and rotor power factor are taken for the given conditions.

Starting Torque (Ts)

Let,

E2 = rotor e.m.f. per phase at standstill
X2 = rotor reactance per phase at standstill
R2 = rotor resistance per phase

Rotor impedance/phase, $Z_2 = \sqrt{R_2^2 + X_2^2}$ ...at standstill

Rotor current/phase, $I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$ ...at standstill

Rotor p.f., $\cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$ ...at standstill

$\therefore$ Starting torque, $T_s = K E_2 I_2 \cos\phi_2$

$$= K E_2 \times \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

$$= \frac{K E_2^2 R_2}{R_2^2 + X_2^2}$$

Generally, the stator supply voltage V is constant so that flux per pole set up by the stator is also fixed. This in turn means that e.m.f. E2 induced in the rotor will be constant.

$$\therefore \quad T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} = \frac{K_1 R_2}{Z_2^2}$$

where K_1 is another constant.

It is clear that the magnitude of starting torque would depend upon the relative values of R_2 and X_2 i.e., rotor resistance/phase and standstill rotor reactance/phase.

It can be shown that $K = 3/2 \pi N_s$.

$$\therefore \quad T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

Note that here N_s is in r.p.s.

Condition for Maximum Starting Torque

It can be proved that starting torque will be maximum when rotor resistance/phase is equal to standstill rotor reactance/phase.

Now $$T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} \qquad \text{(i)}$$

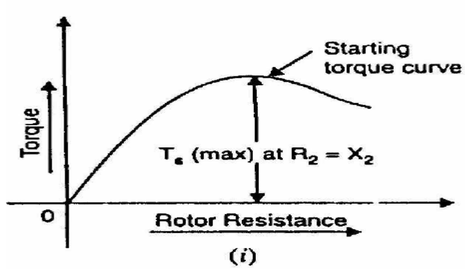
Differentiating eq. (i) w.r.t. R_2 and equating the result to zero, we get,

$$\frac{dT_s}{dR_2} = K_1\left[\frac{1}{R_2^2 + X_2^2} - \frac{R_2(2R_2)}{\left(R_2^2 + X_2^2\right)^2}\right] = 0$$

or $$R_2^2 + X_2^2 = 2R_2^2$$

or $$R_2 = X_2$$

Hence starting torque will be maximum when:
Rotor resistance/phase = Standstill rotor reactance/phase
Under the condition of maximum starting torque, $\Theta_2 = 45°$ and rotor power factor is 0.707 lagging.



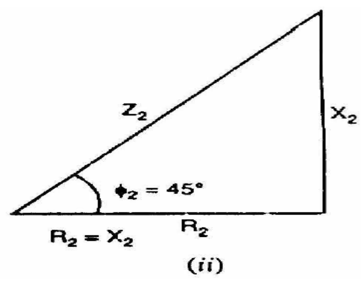


Fig. 3.14 shows the variation of starting torque with rotor resistance. As the rotor resistance is increased from a relatively low value, the starting torque increases until it becomes maximum when R2 = X2. If the rotor resistance is increased beyond this optimum value, the starting torque will decrease.

Effect of Change of Supply Voltage

$$T_s = \frac{K E_2^2 R_2}{R_2^2 + X_2^2}$$

Since $E_2 \propto$ Supply voltage V

$$\therefore \quad T_s = \frac{K_2 V^2 R_2}{R_2^2 + X_2^2}$$

where K_2 is another constant.

$$\therefore \quad T_s \propto V^2$$

Therefore, the starting torque is very sensitive to changes in the value of supply voltage. For example, a drop of 10% in supply voltage will decrease the starting torque by about 20%. This could mean the motor failing to start if it cannot produce a torque greater than the load torque plus friction torque.

Performance Characteristics of Three phase Induction Motor

The equivalent circuits derived in the preceding section can be used to predict the performance characteristics of the induction machine. The important performance characteristics in the steady state are the efficiency, power factor, current, starting torque, maximum (or pull-out) torque.

The complete torque-speed characteristic

In order to estimate the speed torque characteristic let us suppose that a sinusoidal voltage is impressed on the machine. Recalling that the equivalent circuit is the per-phase representation of the machine, the current drawn by the circuit is given by.

$$I_s = \frac{V_s}{(R_s + \frac{R'_r}{s}) + j(X_{ls} + X'_{lr})}$$

Where, Vs is the phase voltage phasor and Is is the current phasor. The magnetizing current is neglected. Since this current is flowing through R′r/s, the air-gap power is given by.

$$\begin{aligned} P_g &= |I_s|^2 \frac{R'_r}{s} \\ &= \frac{V_s}{(R_s + \frac{R'_r}{s})^2 + (X_{ls} + X'_{lr})^2} \frac{R'_r}{s} \end{aligned}$$

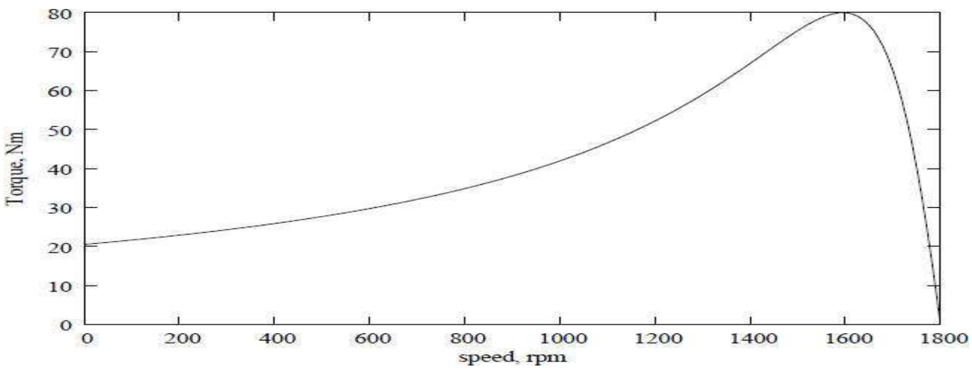
The mechanical power output was shown to be (1−s)Pg (power dissipated in R′r/s). The torque is obtained by dividing this by the shaft speed .Thus we have,

$$\frac{P_g(1-s)}{\omega_m} = \frac{P_g(1-s)}{\omega_s(1-s)} = |I_s|^2 \frac{R_r'}{s\omega_s}$$

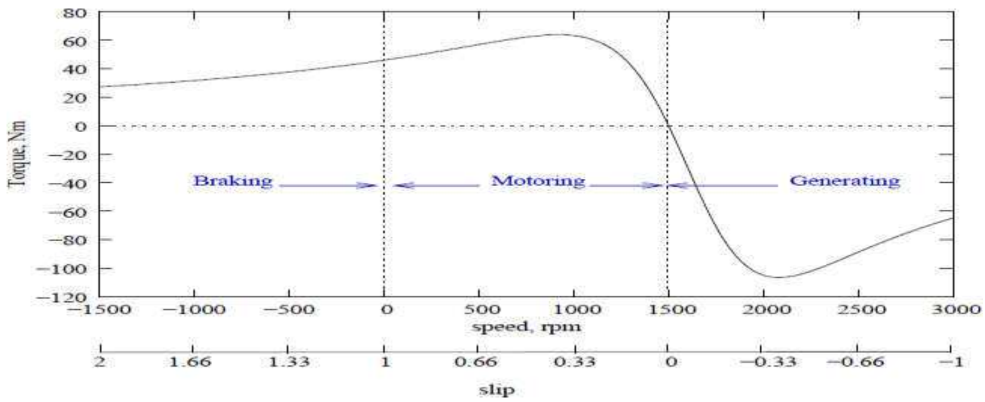
where ω_m is the synchronous speed in radians per second and s is the slip. Further, this is the torque produced per phase. Hence the overall torque is given by

$$T_e = \frac{3}{\omega_s} \cdot \frac{V_s^2}{(R_s + \frac{R_r'}{s})^2 + (X_{ls} + X_{lr}')} \cdot \frac{R_r'}{s}$$

The torque may be plotted as a function of 's' and is called the torque-slip (or torque- speed, since slip indicates speed) characteristic a very important characteristic of the induction machine.
A typical torque-speed characteristic is shown in Fig: 3.18. This plot corresponds to a 3 kW, 4 pole, and 60 Hz machine. The rated operating speed is 1780 rpm.
Further, this curve is obtained by varying slip with the applied voltage being held constant. Coupled with the fact that this is an equivalent circuit valid under steady state, it implies that if this characteristic is to be measured experimentally, we need to look at the torque for a given speed after all transients have died down. One cannot, for example, try to obtain this curve by directly starting the motor with full voltage applied to the terminals and measuring the torque and speed dynamically as it runs up to steady speed.



With respect to the direction of rotation of the air-gap flux, the rotor maybe driven to higher speeds by a prime mover or may also be rotated in the reverse direction. The torque-speed relation for the machine under the entire speed range is called the complete speed-torque characteristic. A typical curve is shown in Fig: 3.19 for a four-pole machine, the synchronous speed being 1500 rpm. Note that negative speeds correspond to slip values greater than 1, and speeds greater than 1500 rpm correspond to negative slip. The plot also shows the operating modes of the induction machine in various regions. The slip axis is also shown for convenience.



Effect of Rotor Resistance on Speed Torque Characteristic

Restricting ourselves to positive values of slip, we see that the curve has a peak point. This is the maximum torque that the machine can produce, and is called as stalling torque. If the load torque is more than this value, the machine stops rotating or stalls. It occurs at a slip ˆs, which for the machine of Fig: 3.19 is 0.38. At values of slip lower than ˆs, the curve falls steeply down to zero at s = 0. The torque at synchronous speed is therefore zero. At values of slip higher than s = ˆs, the curve falls slowly to a minimum value at s = 1. The torque at s = 1 (speed = 0) is called the starting torque. The value of the stalling torque may be obtained by differentiating the expression for torque with respect to zero and setting it to zero to find the value of ˆs. Using this method, we can write

$$\hat{s} = \frac{\pm R'_r}{\sqrt{R'^2_r + (X_{ls} + X'_{lr})^2}}$$

– Substituting ˆs into the expression for torque gives us the value of the stalling torque ˆ Te,

$$\hat{T}_e = \frac{3V_s^2}{2\omega_s} \cdot \frac{1}{R_s \pm \sqrt{R_s^2 + (X_{ls} + X'_{lr})^2}}$$

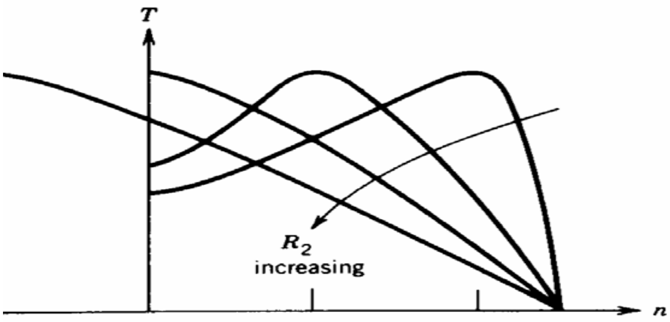
The negative sign being valid for negative slip.

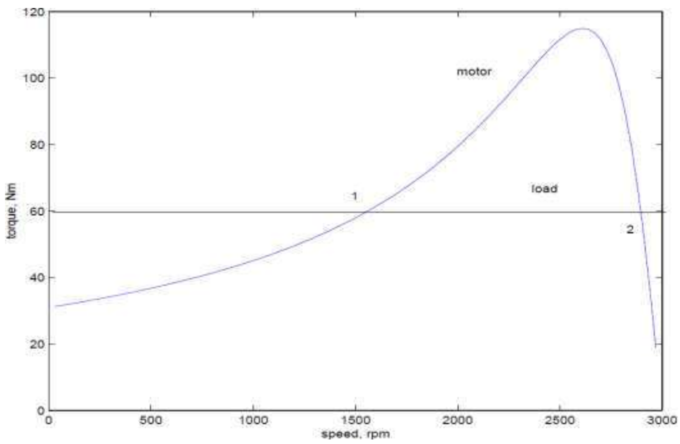
The expression shows that ˆ Te is the independent of R′r, while ˆs is directly proportional to R′r. This fact can be made use of conveniently to alter ˆs. If it is possible to change R′r, then we can get a whole series of torque-speed characteristics, the maximum torque remaining constant all the while.

We may note that if R′r is chosen equal to =

$$\sqrt{R_s^2 + (X_{ls} + X'_{lr})^2}$$

The ˆs, becomes unity, which means that the maximum torque occurs at starting. Thus changing of R′r, wherever possible can serve as a means to control the starting torque Fig:3.20.

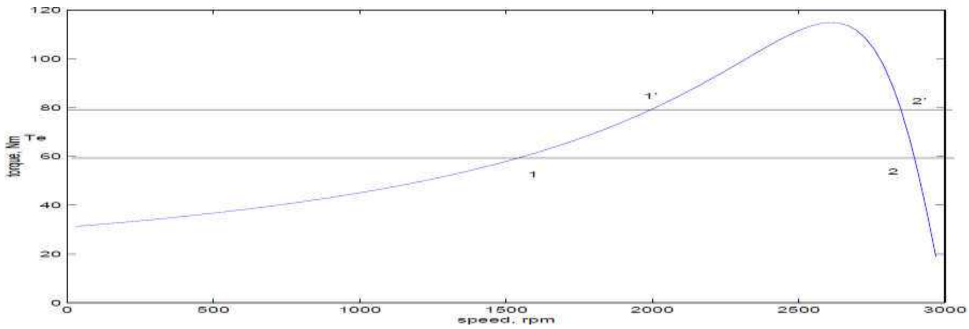




The system consisting of the motor and load will operate at a point where the two characteristics meet. From the above plot, we note that there are two such points. We therefore need to find out which of these is the actual operating point. To answer this we must note that, in practice, the characteristics are never fixed; they change slightly with time. It would be appropriate to consider a small band around the curve drawn where the actual points of the characteristic will lie. This being the case let us considers that the system is operating at point 1, and the load torque demand increases slightly. This is shown in Fig: 3.22, where the change is exaggerated for clarity. This would shift the point of operation to a point 1′ at which the slip would be less and the developed torque higher.

The difference in torque developed ΔTe, being positive will accelerate the machine. Any overshoot in speed as it approaches the point 1′ will cause it to further accelerate since the developed torque is increasing. Similar arguments may be used to show that if for some reason the developed torque becomes smaller the speed would drop and the effect is cumulative. Therefore we may conclude that 1 is not a stable operating point.

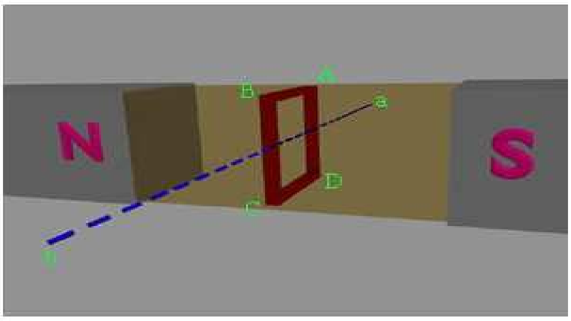
Let us consider the point 2. If this point shifts to 2′, the slip is now higher (speed is lower) and the positive difference in torque will accelerate the machine. This behaviour will tend to bring the operating point towards 2 once again. In other words, disturbances at point 2 will not cause a runaway effect. Similar arguments may be given for the case where the load characteristic shifts down. Therefore we conclude that point 2 is a stable operating point.



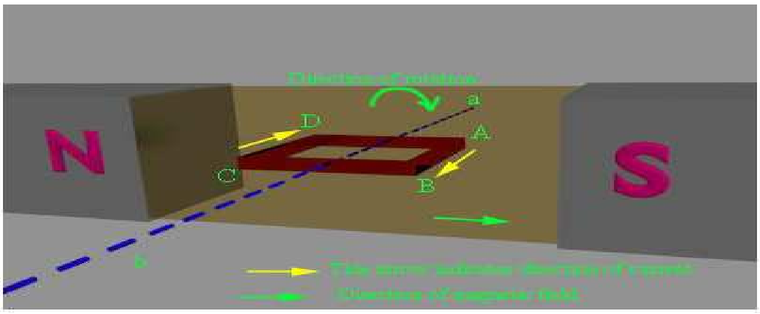
From the above discussions, we can say that the entire region of the speed-torque characteristic from s = 0 to s = ˆs is an unstable region, while the region from s = ˆs to s = 0 is a stable region. Therefore the machine will always operate between s = 0 and s = ˆs.

AC MACHINES

Working Principle of Alternator

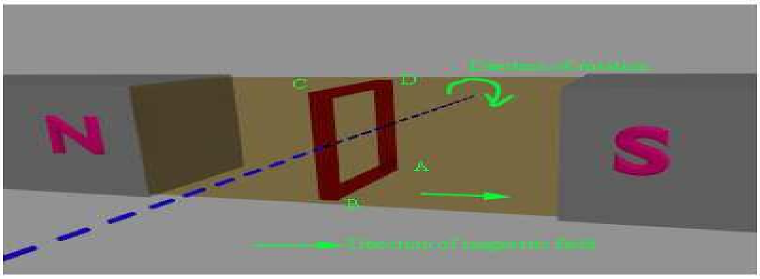


The working principle of alternator is very simple. It is just like basic principle of DC generator. It also depends upon Faraday's law of electromagnetic induction which says the current is induced in the conductor inside a magnetic field when there is a relative motion between that conductor and the magnetic field. For understanding working of alternator let's think about a single rectangular turn placed in between two opposite magnetic pole as shown above.



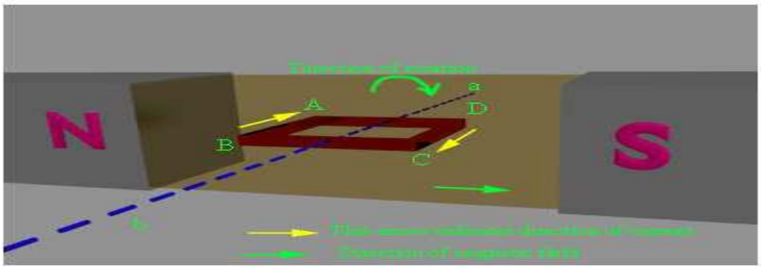
Say this single turn loop ABCD can rotate against axis a-b. Suppose this loop starts rotating clockwise. After 90o rotation the side AB or conductor AB of the loop comes in front of S-pole and conductor CD comes in front of N-pole. At this position the tangential motion of the conductor AB is just perpendicular to the magnetic flux lines from N to S pole. Hence rate of flux cutting by the conductor AB is maximum here and for that flux cutting there will be an induced current in the conductor AB and direction of the induced current can be determined by Flemming's right hand rule. As per this rule the direction of this current will be from A to B. At the same time conductor CD comes under N pole and here also if we apply Fleming right hand rule we will get the direction of induced current and it will be from C to D.

Now after clockwise rotation of another 90o the turn ABCD comes at vertical position as shown below. At this position tangential motion of conductor AB and CD is just parallel to the magnetic flux lines, hence there will be no flux cutting that is no current in the conductor. While the turn ABCD comes from horizontal position to vertical position, angle between flux lines and direction of motion of conductor, reduces from 90o to 0o and consequently the induced current in the turn is reduced to zero from its maximum value.



After another clockwise rotation of 90o the turn again come to horizontal position and here conductor AB comes under N-pole and CD comes under S-pole, and here if we again apply

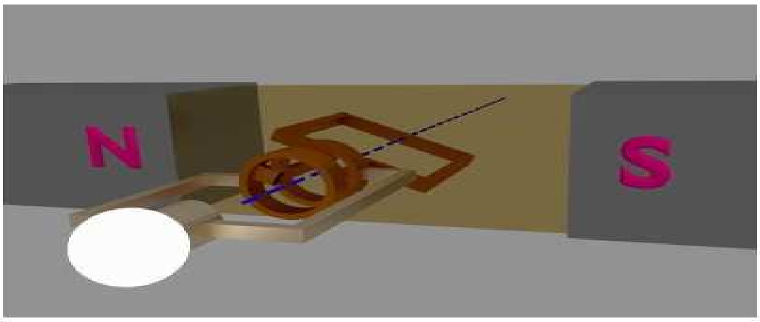
Flemming's right hand rule, we will see that induced current in conductor AB, is from point B to A and induced current in the conductor CD is from D to C.



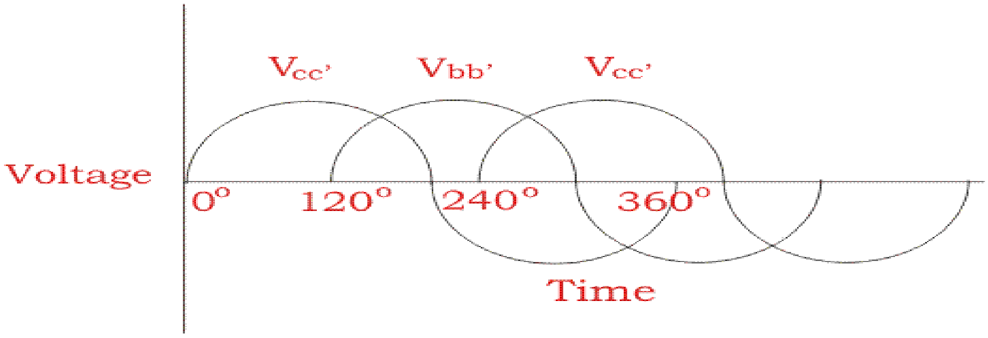
As at this position the turn comes at horizontal position from its vertical position, the current in the conductors comes to its maximum value from zero. That means current is circulating in the close turn from point B to A, from A to D, from D to C and from C to B. Just reverse of the previous horizontal position when the current was circulating as A → B → C → D → A.

While the turn further proceeds to its vertical position the current is again reduced to zero. So if the turn continues to rotate the current in the turn continually alternate its direction. During every full revolution of the turn, the current in the turn gradually reaches to its maximum value then reduces to zero and then again it comes to its maximum value but in opposite direction and again it comes to zero. In this way the current completes one full sine wave form during each 360o revolution of the turn. So we have seen how an alternating current is produced in a turn is rotated inside a magnetic field. From this, we will now come to the actual working principle of alternator.

Now we cut the loop and connect its two ends with two slip rings and stationary brush is placed on each slip ring. If we connect two terminals of an external load with these two brushes, we will get an alternating current in the load. This is our elementary model of alternator.



Having understood the very basic principle of alternator, let us now have an insight into its basic operational principal of a practical alternator. During discussion of basic **working of** ^{o}, we have considered that the magnetic field is stationary and conductors (armature) is rotating. But generally in practical construction of alternator, armature conductors are stationary and field magnets rotate between them. The rotor of an alternator or a synchronous generator is mechanically coupled to the shaft or the turbine blades, which on being made to rotate at synchronous speed N_s under some mechanical force results in magnetic flux cutting of the stationary armature conductors housed on the stator. As a direct consequence of this flux cutting an induced emf and current starts to flow through the armature conductors which first flow in one direction for the first half cycle and then in the other direction for the second half cycle for each winding with a definite time lag of 120^o due to the space displaced arrangement of 120^o between them as shown in the figure below. This particular phenomena results in 3φ power flow out of the alternator which is then transmitted to the distribution stations for domestic and industrial uses.



3 phase generated voltage

Synchronous Impedance Method or E.M.F. Method

The method is also called E.M.F. method of determining the regulation. The method requires following data to calculate the regulation.

1. The armature resistance per phase (Ra).
2. Open circuit characteristics which is the graph of open circuit voltage against the field current. This is possible by conducting open circuit test on the alternator.
3. Short circuit characteristics which is the graph of short circuit current against field current. This is possible by conducting short circuit test on the alternator.

Let us see, the circuit diagram to perform open circuit as well as short circuit test on the alternator. The alternator is coupled to a prime mover capable of driving the alternator at its synchronous speed. The armature is connected to the terminals of a switch. The other terminals of the switch are short circuited through an ammeter. The voltmeter is connected across the lines to measure the open circuit voltage of the alternator.

The field winding is connected to a suitable d.c. supply with rheostat connected in series. The field excitation i.e. field current can be varied with the help of this rheostat. The circuit diagram is shown in the Fig. 1.

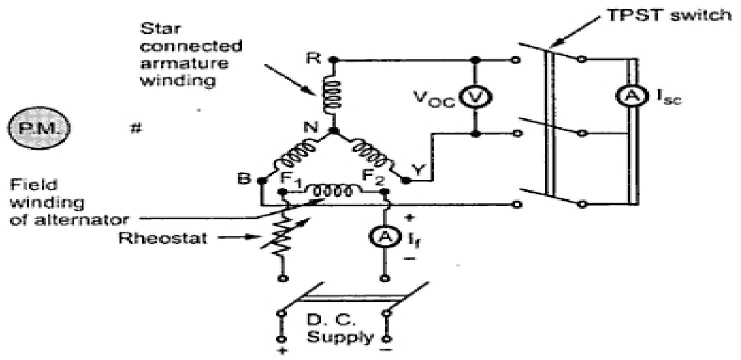


Fig. 1 Circuit diagram for open circuit and short circuit test on alternator

Open Circuit Test

Procedure to conduct this test is as follows :

i) Start the prime mover and adjust the speed to the synchronous speed of the alternator.

ii) Keeping rheostat in the field circuit maximum, switch on the d.c. supply.

iii) The T.P.S.T switch in the armature circuit is kept open.

iv) With the help of rheostat, field current is varied from its minimum value to the rated value. Due to this, flux increasing the induced e.m.f. Hence voltmeter reading, which is measuring line value of open circuit voltage increases. For various values of field current, voltmeter readings are observed.

The observation for open circuit test are tabulated as below :

Sr. No.	I_f A	V_{oc} (line) V	V_{oc} (phase) = V_{oc} (line)/$\sqrt{3}$ V
1			
2			
:			
:			

From the above table, graph of (Voc)ph against If is plotted.

Note : This is called open circuit characteristics of the alternator, called O.C.C. This is shown in the Fig. 2.

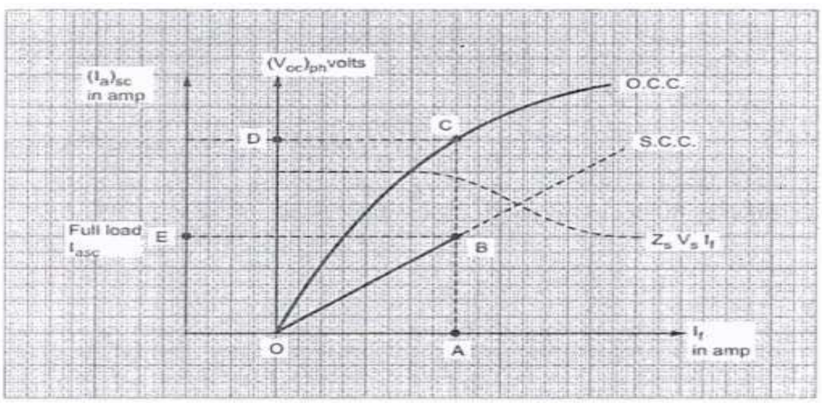


Fig. 2 O.C.C. and S.C.C. of an alternator

Short Circuit Test

After completing the open circuit test observation, the field rheostat is brought to maximum position, reducing field current to a minimum value. The T.P.S.T switch is closed. As ammeter has negligible resistance, the armature gets short circuited. Then the field excitation is gradually increased till full load current is obtained through armature winding. This can be observed on the ammeter connected in the armature circuit. The graph of short circuit armature current against field current is plotted from the observation table of short circuit test. This graph is called short circuit characteristics, S.C.C. This is also shown in the Fig. 2.

Observation table for short circuit test :

Sr. No.	I_f A	Short circuit armature current per phase (I_{asc}) A
1		
2		

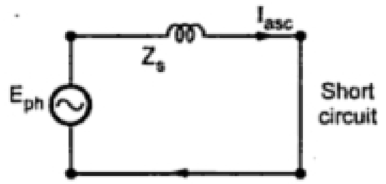
The S.C.C. is a straight line graph passing through the origin while O.C.C. resembles B-H curve of a magnetic material.

Note: As S.C.C. is straight line graph, only one reading corresponding to full load armature current along with the origin is sufficient to draw the straight line.

1.3 Determination of From O.C.C. and S.C.C.

The synchronous impedance of the alternator changes as load condition changes. O.C.C. and S.C.C. can be used to determine Zs for any load and load p.f. conditions.

In short circuit test, external load impedance is zero. The short circuit armature current is circulated against the impedance of the armature winding which is Zs. The voltage responsible for driving this short circuit current is internally induced e.m.f. This can be shown in the equivalent circuit drawn in the Fig. 3.



Equivalent circuit on short circuit

From the equivalent circuit we can write,

$$Z_s = E_{ph} / I_{asc}$$

Now value of Iasc is known, which can observed on the alternator. But internally induced e.m.f. can not be observed under short circuit condition. The voltmeter connected will read zero which is voltage across short circuit. To determine Zs it is necessary to determine value of E which is driving Iasc against Zs.

Now internally induced e.m.f. is proportional to the flux i.e. field current If.

Eph α Φ α If ...... from e.m.f. equation

So if the terminal of the alternator are opened without disturbing If which was present at the time of short circuited condition, internally induced e.m.f. will remain same as Eph. But now current will be zero. Under this condition equivalent circuit will become as shown in the Fig. 4.

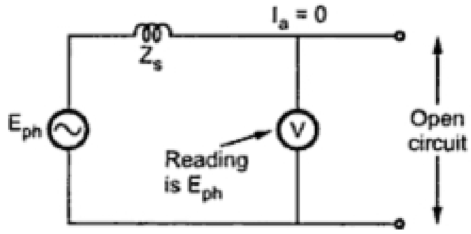


Fig. 4

It is clear now from the equivalent circuit that as Ia = 0 the voltmeter reading (Voc)ph will be equal to internally induced e.m.f. (Eph).

$$E_{ph} = (V_{oc})_{ph} \text{ on open circuit}$$

This is what we are interested in obtaining to calculate value of Zs. So expression for Zs can be modified as,

$$Z_s = \left.\frac{(V_{oc})_{ph}}{(I_{asc})_{ph}}\right|_{\text{for same } I_f}$$

Thus in general,

$$Z_s = \left.\frac{\text{Phase e.m.f. on open circuit}}{\text{Phase current on short circuit}}\right|_{\text{For same excitation current}}$$

So O.C.C. and S.C.C. can be effectively to calculate Zs.

The value of Zs is different for different values of If as the graph of O.C.C. is non linear in nature.

So suppose Zs at full load is required then,

Iasc = full load current.

From S.C.C. determine If required to drive this full load short circuit Ia. This is equal to 'OA', as shown in the Fig.2.

Now for this value of If, (Voc)ph can be obtained from O.C.C. Extend kine from point A, till it meets O.C.C. at point C. The corresponding (Voc)ph value is available at point D.

(Voc)ph = OD

While (Iasc)ph = OE

$$\therefore \quad Z_s \text{ at full load} = \left.\frac{(V_{oc})_{ph}}{\text{Full load }(I_{asc})_{ph}}\right|_{\text{same } I_f \text{ (same excitation)}}$$

$$= \left.\frac{OD}{OE}\right|_{\text{same } I_f = OA}$$

at full load

General steps to determine Zs at any load condition are :

i) Determine the value of (Iasc)ph for corresponding load condition. This can be determined from known full load current of the alternator. For half load, it is half of the full load value and so on.

ii) S.C.C. gives relation between (Iasc)ph and If. So for (Iasc)ph required, determine the corresponding value of If from S.C.C.

iii) Now for this same value of If, extend the line on O.C.C. to get the value of (Voc)ph. This is (Voc)ph for same If, required to drive the selected (Iasc)ph.

iv) The ratio of (Voc)ph and (Iasc)ph, for the same excitation gives the value of Zs at any load conditions.

The graph of synchronous impedance against excitation current is also shown in the Fig. 2.

1.4 Regulation Calculations

From O.C.C. and S.C.C., Zs can be determined for any load condition.

The armature resistance per phase (Ra) can be measured by different methods. One of the method is applying d.c. known voltage across the two terminals and measuring current. So value of Ra per phase is known.

$$\text{Now} \qquad Z_s = \sqrt{(R_a)^2 + (X_s)^2}$$

$$\boxed{X_s = \sqrt{(Z_s)^2 - (R_a)^2}\ \Omega/\text{ph}}$$

So synchronous reactance per phase can be determined.

No load induced e.m.f. per phase, Eph can be determined by the mathematical expression derived earlier.

$$\boxed{E_{ph} = \sqrt{(V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi \pm I_a X_s)^2}}$$

where Vph = Phase value of rated voltage

Ia = Phase value of current depending on the load condition

cosΦ = p.f. of load

Positive sign for lagging power factor while negative sign for leading power factor, Ra and Xs values are known from the various tests performed.

The regulation then can be determined by using formula,

$$\% \text{ Regulation} = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100$$

Advantages and Limitations of Synchronous Impedance Method

- The main advantages of this method are the value of synchronous impedance Zs for any load condition can be calculated. Hence regulation of the alternator at any load condition and load power factor can be determined. Actual load need not be connected to the alternator and hence method can be used for very high capacity alternators.

- The main limitation of this method is that the method gives large values of synchronous reactance. This leads to high values of percentage regulation than the actual results. Hence this method is called pessimistic method.

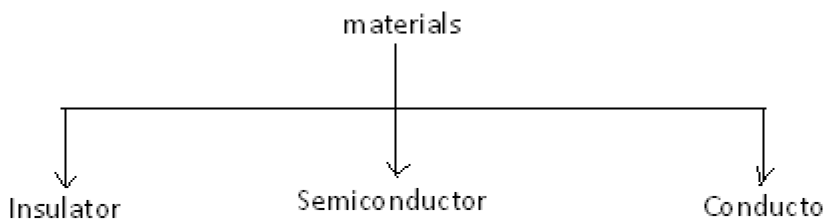
UNIT –IV

DIODES

PN JUNCTION DIODE

1.0 INTRODUCTON

Based on the electrical conductivity all the materials in nature are classified as insulators, semiconductors, and conductors.



Insulator: An insulator is a material that offers a very low level (or negligible) of conductivity when voltage is applied. Eg: Paper, Mica, glass, quartz. Typical resistivity level of an insulator is of the order of 10^{10} to 10^{12} **Ω**-cm. The energy band structure of an insulator is shown in the fig.1.1. Band structure of a material defines the band of energy levels that an electron can occupy. Valance band is the range of electron energy where the electron remain bended too the atom and do not contribute to the electric current. Conduction bend is the range of electron energies higher than valance band where electrons are free to accelerate under the influence of external voltage source resulting in the flow of charge.

The energy band between the valance band and conduction band is called as forbidden band gap. It is the energy required by an electron to move from balance band to conduction band i.e. the energy required for a valance electron to become a free electron.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

For an insulator, as shown in the fig.1.1 there is a large forbidden band gap of greater than 5Ev. Because of this large gap there a very few electrons in the CB and hence the conductivity of insulator is poor. Even an increase in temperature or applied electric field is insufficient to transfer electrons from VB to CB.

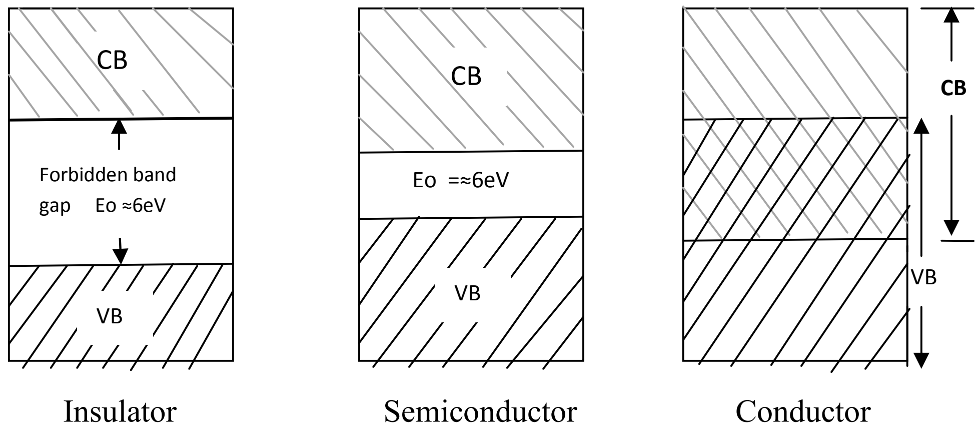


Fig: 1.1 Energy band diagrams insulator, semiconductor and conductor

Conductors: A conductor is a material which supports a generous flow of charge when a voltage is applied across its terminals. i.e. it has very high conductivity. Eg: Copper, Aluminum, Silver, Gold. The resistivity of a conductor is in the order of 10^{-4} and 10^{-6} Ω-cm. The Valance and conduction bands overlap (fig1.1) and there is no energy gap for the electrons to move from valance band to conduction band. This implies that there are free electrons in CB even at absolute zero temperature (0K). Therefore at room temperature when electric field is applied large current flows through the conductor.

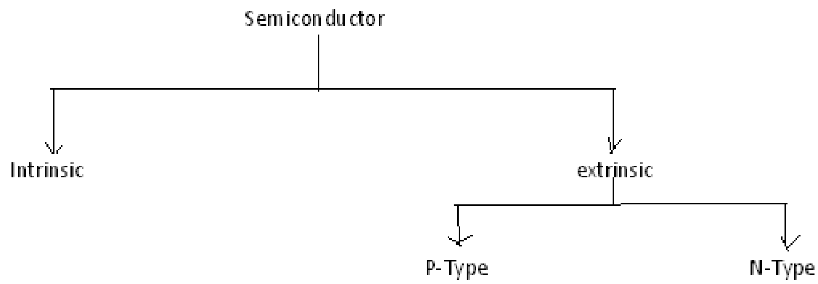
Semiconductor: A semiconductor is a material that has its conductivity somewhere between the insulator and conductor. The resistivity level is in the range of 10 and 10^4 Ω-cm. Two of the most commonly used are Silicon (Si=14 atomic no.) and germanium (Ge=32 atomic no.). Both have 4 valance electrons. The forbidden band gap is in the order of 1eV. For eg., the band gap energy for Si, Ge and GaAs is 1.21, 0.785 and 1.42 eV, respectively at absolute zero temperature (0K). At 0K and at low temperatures, the valance band electrons do not have sufficient energy to move from V to CB. Thus semiconductors act a insulators at 0K. as the temperature increases, a large number of valance electrons acquire sufficient energy to leave the VB, cross the forbidden bandgap and reach CB. These are now free electrons as they can move freely under the influence of electric field. At room temperature there are sufficient electrons in the CB and hence the semiconductor is capable of conducting some current at room temperature.

Inversely related to the conductivity of a material is its resistance to the flow of charge or current. Typical resistivity values for various materials' are given as follows.

Insulator	Semiconductor	Conductor
10^{-6} Ω-cm (Cu)	50Ω-cm (Ge)	10^{12} Ω-cm (mica)
	50x10^3 Ω-cm (Si)	

Typical resistivity values

1.0.1 SEMICONDUCTOR TYPES



A pure form of semiconductors is called as intrinsic semiconductor. Conduction in intrinsic sc is either due to thermal excitation or crystal defects. Si and Ge are the two most important semiconductors used. Other examples include Gallium arsenide GaAs, Indium Antimonide (InSb) etc.

Let us consider the structure of Si.A Si atomic no. is 14 and it has 4 valance electrons. These 4 electrons are shared by four neighboring atoms in the crystal structure by means of covalent bond. Fig. 1.2a shows the crystal structure of Si at absolute zero temperature (0K). Hence a pure SC acts has poor conductivity (due to lack of free electrons) at low or absolute zero temperature.

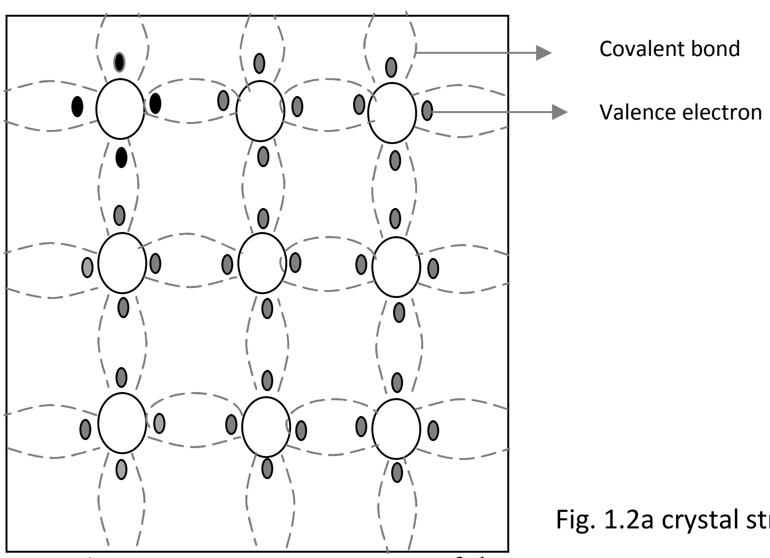


Fig. 1.2a crystal structure of Si at 0K

At room temperature some of the covalent bonds break up to thermal energy as shown in fig 1.2b. The valance electrons that jump into conduction band are called as free electrons that are available for conduction.

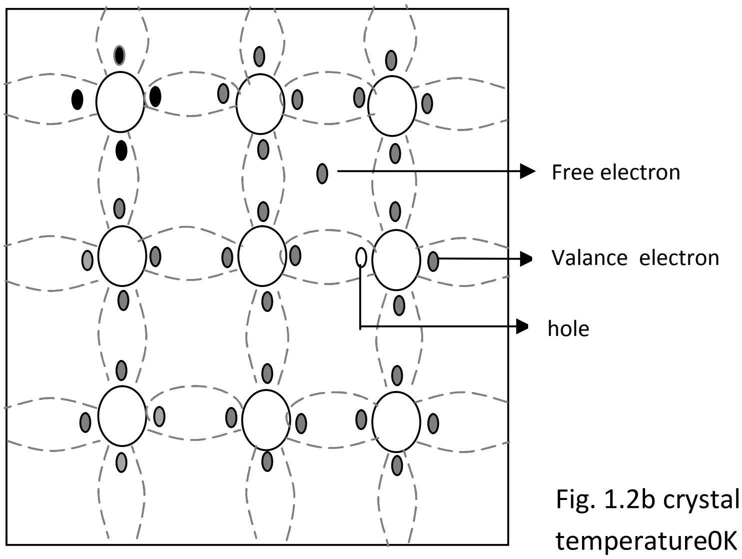


Fig. 1.2b crystal structure of Si at room temperature0K

The absence of electrons in covalent bond is represented by a small circle usually referred to as hole which is of positive charge. Even a hole serves as carrier of electricity in a manner similar to that of free electron.

The mechanism by which a hole contributes to conductivity is explained as follows:

When a bond is in complete so that a hole exists, it is relatively easy for a valance electron in the neighboring atom to leave its covalent bond to fill this hole. An electron moving from a bond to fill a hole moves in a direction opposite to that of the electron. This hole, in its new position may now be filled by an electron from another covalent bond and the hole will correspondingly move one more step in the direction opposite to the motion of electron. Here we have a mechanism for conduction of electricity which does not involve free electrons. This phenomenon is illustrated in fig1.3

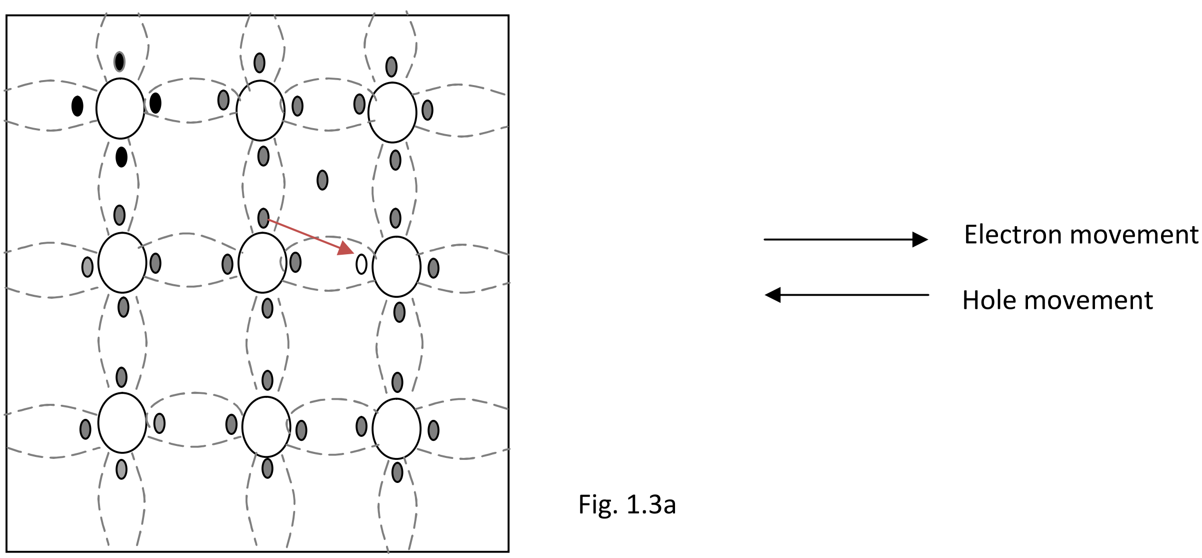


Fig. 1.3a

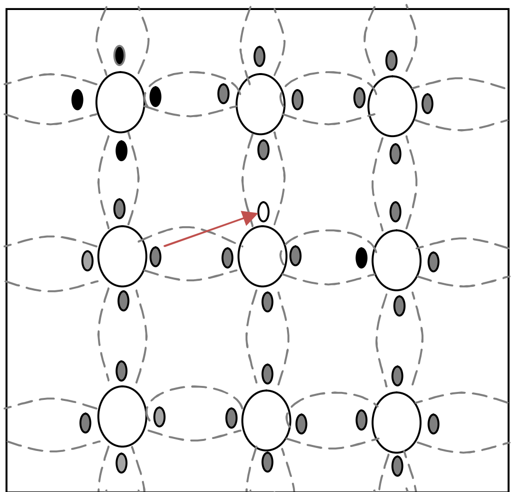


Fig. 1.3b

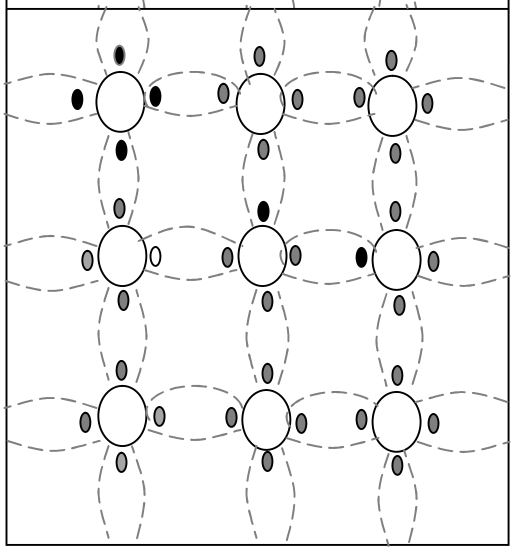


Fig. 1.3c

Fig 1.3a show that there is a hole at ion 6.Imagine that an electron from ion 5 moves into the hole at ion 6 so that the configuration of 1.3b results. If we compare both fig1.3a &fig 1.3b, it appears as if the hole has moved towards the left from ion6 to ion 5. Further if we compare fig 1.3b and fig 1.3c, the hole moves from ion5 to ion 4. This discussion indicates the motion of hole is in a direction opposite to that of motion of electron. Hence we consider holes as physical entities whose movement constitutes flow of current.

In a pure semiconductor, the number of holes is equal to the number of free electrons.

1.0.2 EXTRINSIC SEMICONDUCTOR:

Intrinsic semiconductor has very limited applications as they conduct very small amounts of current at room temperature. The current conduction capability of intrinsic semiconductor can be increased significantly by adding a small amounts impurity to the intrinsic semiconductor. By adding impurities it becomes impure or extrinsic semiconductor. This process of adding impurities is called as doping. The amount of impurity added is 1 part in 10^6 atoms.

N type semiconductor: If the added impurity is a pentavalent atom then the resultant semiconductor is called N-type semiconductor. Examples of pentavalent impurities are Phosphorus, Arsenic, Bismuth, Antimony etc.

A pentavalent impurity has five valance electrons. Fig 1.3a shows the crystal structure of N-type semiconductor material where four out of five valance electrons of the impurity atom(antimony) forms covalent bond with the four intrinsic semiconductor atoms. The fifth electron is loosely bound to the impurity atom. This loosely bound electron can be easily

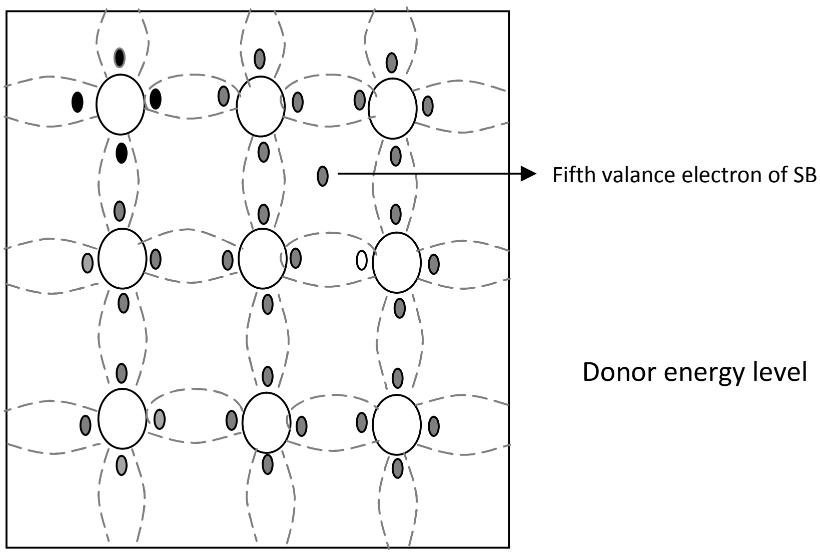


Fig. 1.3a crystal structure of N type SC

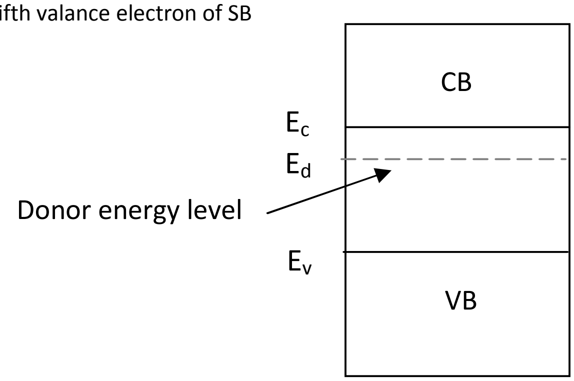


Fig. 1.3bEnergy band diagram of N type

excited from the valance band to the conduction band by the application of electric field or increasing the thermal energy. The energy required to detach the fifth electron form the impurity atom is very small of the order of 0.01ev for Ge and 0.05 eV for Si.

The effect of doping creates a discrete energy level called donor energy level in the forbidden band gap with energy level E_d slightly less than the conduction band (fig 1.3b). The difference between the energy levels of the conducting band and the donor energy level is the energy required to free the fifth valance electron (0.01 eV for Ge and 0.05 eV for Si). At room temperature almost all the fifth electrons from the donor impurity atom are raised to conduction band and hence the number of electrons in the conduction band increases significantly. Thus every antimony atom contributes to one conduction electron without creating a hole.

In the N-type sc the no. of electrons increases and the no. of holes decreases compared to those available in an intrinsic sc. The reason for decrease in the no. of holes is that the larger no.

of electrons present increases the recombination of electrons with holes. Thus current in N type sc is dominated by electrons which are referred to as majority carriers. Holes are the minority carriers in N type sc

P type semiconductor: If the added impurity is a trivalent atom then the resultant semiconductor is called P-type semiconductor. Examples of trivalent impurities are Boron, Gallium , indium etc.

The crystal structure of p type sc is shown in the fig1.3c. The three valance electrons of the impurity (boon) forms three covalent bonds with the neighboring atoms and a vacancy exists in the fourth bond giving rise to the holes. The hole is ready to accept an electron from the neighboring atoms. Each trivalent atom contributes to one hole generation and thus introduces a large no. of holes in the valance band. At the same time the no. electrons are decreased compared to those available in intrinsic sc because of increased recombination due to creation of additional holes.

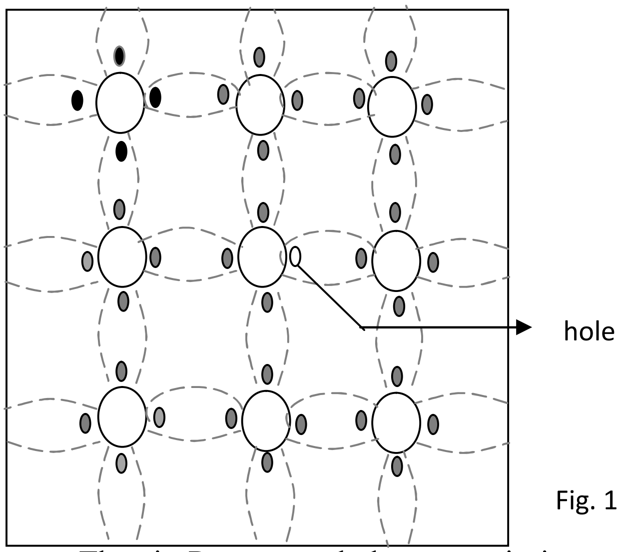


Fig. 1.3c crystal structure of P type sc

Thus in P type sc , holes are majority carriers and electrons are minority carriers. Since each trivalent impurity atoms are capable accepting an electron, these are called as acceptor atoms. The following fig 1.3d shows the pictorial representation of P type sc

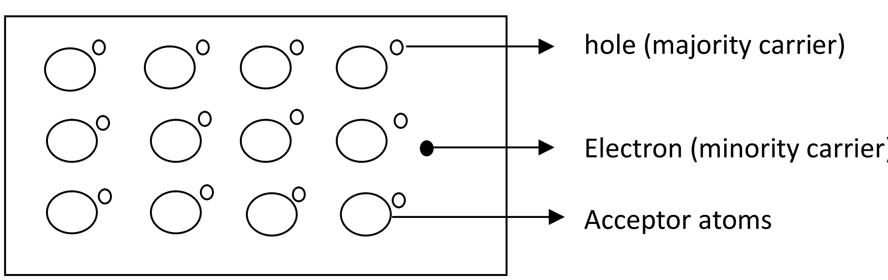


Fig. 1.3d crystal structure of P type sc

- The conductivity of N type sc is greater than that of P type sc as the mobility of electron is greater than that of hole.
- For the same level of doping in N type sc and P type sc, the conductivity of an Ntype sc is around twice that of a P type sc

THEORY OF PN JUNCTION DIODE:

PN JUNCTION DIODE

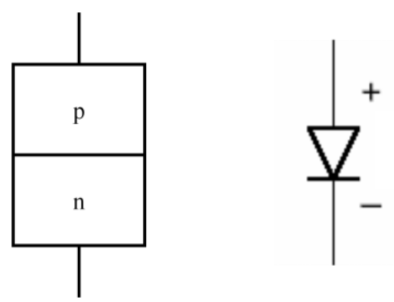
➢ A p–n junction is formed by joining P-type and N-type semiconductors together in very close contact.

➢ The term junction refers to the boundary interface where the two regions of the semiconductor meet.

➢ Diode is a two-terminal electronic component that conducts electric current in only one direction.

➢ The crystal conducts conventional current in a direction from the p-type side (called the anode) to the n-type side (called the cathode), but not in the opposite direction.

Symbol of PN junction diode

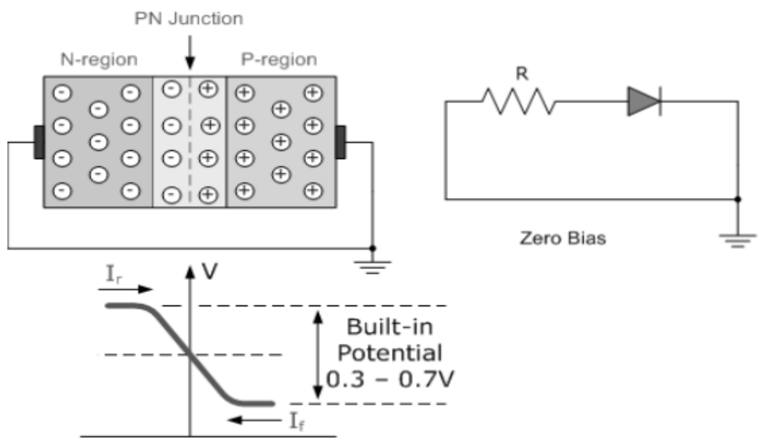


Biasing

"Biasing" is providing minimum external voltage and current to activate the device to study its characteristics.

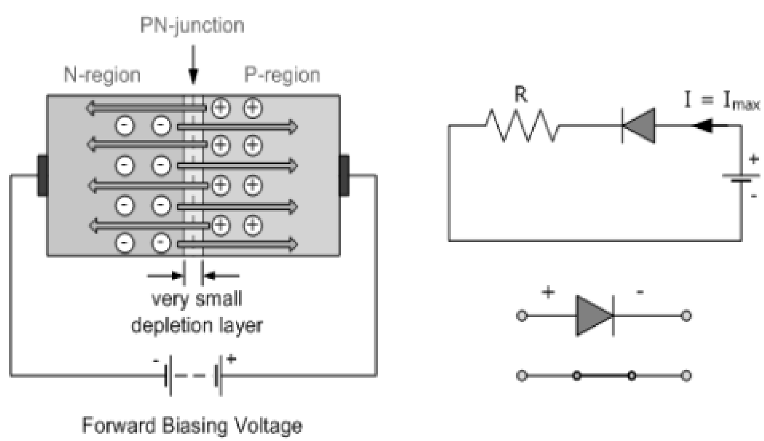
There are two operating regions and two "biasing" conditions for the standard Junction Diode and they are:

❖ **Zero Bias:**



When a diode is **Zero Biased** no external energy source is applied and a natural **Potential Barrier** is developed across a depletion layer.

(i) Forward Bias:



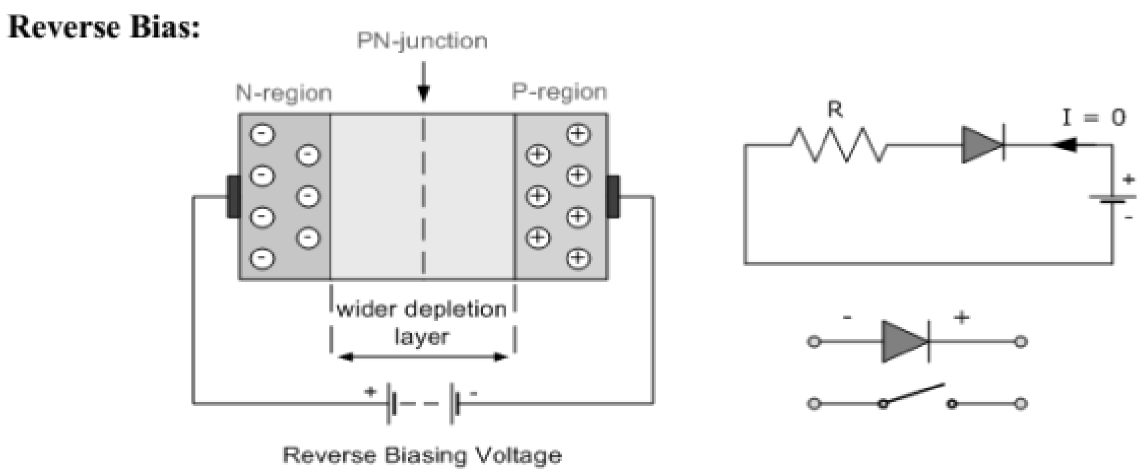
When the positive terminal of a battery is connected to P-type semiconductor and negative terminal to N-type is known as forward bias of PN junction.

➢ The applied forward potential establishes an electric field opposite to the potential barrier Therefore the potential barrier is reduced at the junction. As the potential barrier is very small (0.3V for Ge and 0.7V for Si),a small forward voltage is sufficient to completely eliminate the barrier potential, thus the junction resistance becomes zero.

➢ In other words, the applied positive potential repels the holes in the 'P' region so that the holes moves towards the junction and applied negative potential repels the electrons in the 'N' region towards the junction results in depletion region starts decreasing. When the applied potential is more than the internal barrier potential then the depletion region completely disappear, thus the junction resistance becomes zero.

➢ Once the potential barrier is eliminated by a forward voltage, j unction establishes the low resistance path for the entire circuit, thus a current flows in the circuit, it is called as forward current.

Reverse Bias:



For reverse bias, the negative terminal is connected to P-type semiconductor and positive terminal to N type semiconductor.

➢ When reverse bias voltage is applied to the junction, all the majority carriers of 'P' region are attracted towards the negative terminal of the battery and the majority carriers of the N region attracted towards the positive terminal of the battery, hence the depletion region increases.

➢ The applied reverse voltage establishes an electric field which acts in the same direction of the potential barrier. Therefore, the resultant field at the junction is strengthened and the barrier width is increased. This increased potential barrier prevents the flow of charge carriers across the junction, results in a high resistance path.

➢ This process cannot continue indefinitely because after certain extent the junction break down occurs. As a result a small amount of current flows through it due to minority carriers. This current is known as "reverse saturation current".

V-I characteristics of PN junction diode

Forward Bias:

➢ The application of a forward biasing voltage on the junction diode results in the depletion layer becoming very thin and narrow which represents a low impedance path through the junction thereby allowing high currents to flow.

➢ The point at which this sudden increase in current takes place is represented on the static I-V characteristics curve above as the "knee" point.

Reverse Bias:

➢ In Reverse biasing voltage a high resistance value to the PN junction and practically zero current flows through the junction diode with an increase in bias voltage.
➢ However, a very small leakage current does flow through the junction which can be measured in microamperes, (μA).

➢ One final point, if the reverse bias voltage Vr applied to the diode is increased to a sufficiently high enough value, it will cause the PN junction to overheat and fail due to the avalanche effect around the junction.

➢ This may cause the diode to become shorted and will result in the flow of maximum circuit current, and this shown as a step downward slope in the reverse static characteristics curve below.

VI CHARACTERISTICS AND THEIR TEMPERATURE DEPENDENCE:

Diode terminal characteristics equation for diode junction current:

$$I_D = I_0 (e^{\frac{v}{\eta v_T}} - 1)$$

Where VT = kT/q;

VD_ diode terminal voltage, Volts

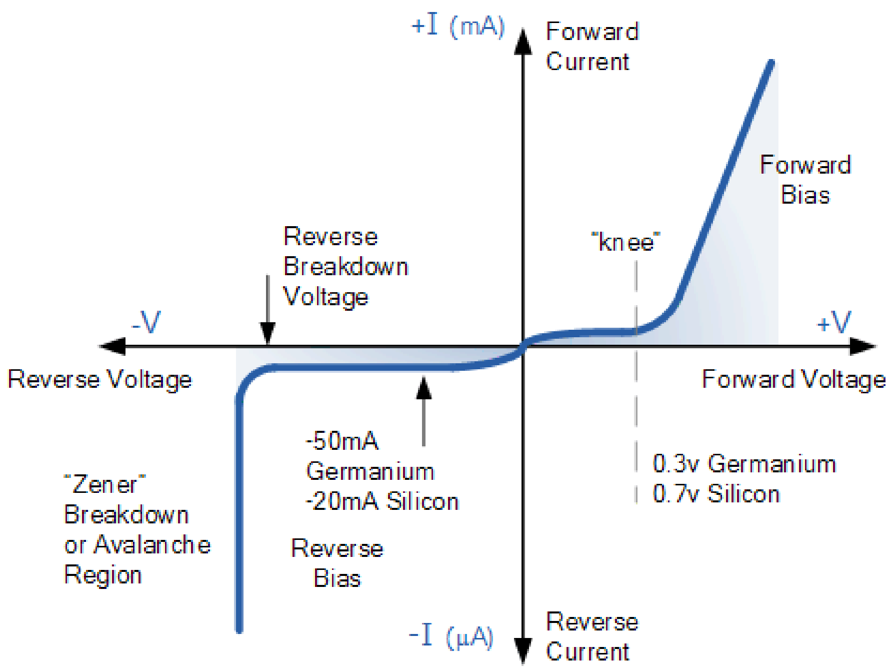
Io _ temperature-dependent saturation current, µA

T _ absolute temperature of p-n junction, K

k _ Boltzmann's constant 1.38x 10 -23J/K)

q _ electron charge 1.6x10-19 C

η = empirical constant, 1 for Ge and 2 for Si



DIFFUSION CAPACITANCE AND SPACE CHARGE CAPACITANCE

Diffusion capacitance

 As a p-n diode is forward biased, the minority carrier distribution in the quasi-neutral region increases dramatically.

 In addition, to preserve quasi-neutrality, the majority carrier density increases by the same amount.

 This effect leads to an additional capacitance called the diffusion capacitance.

 The diffusion capacitance is calculated from the change in charge with voltage:

$$C = \frac{d\Delta Q}{dV_a}$$

 Where the charge, DQ, is due to the excess carriers.

 Unlike a parallel plate capacitor, the positive and negative charge is not spatially separated. Instead, the electrons and holes are separated by the energy band gap.

 Nevertheless, this voltage dependent charges yield a capacitance just as the one associated with a parallel plate capacitor.

 The excess minority-carrier charge is obtained by integrating the charge density over the quasi-neutral region:

$$\Delta Q_p = \int_{x_n}^{w_n} qA(p_n - p_{n0})\, dx$$

Space Charge capacitance:

 After joining p-type and n-type semiconductors, electrons near the p–n interface tend to diffuse into the p region.

 As electrons diffuse, they leave positively charged ions (donors) in the n region.

 Similarly, holes near the p–n interface begin to diffuse into the n-type region leaving fixed ions (acceptors) with negative charge.

 The regions nearby the p–n interfaces lose their neutrality and become charged, forming the space charge capacitance.

CLASSIFICATION OF RECTIFIERS:

Using one or more diodes in the circuit, following rectifier circuits can be designed.

1) Half - Wave Rectifier
2) Full – Wave Rectifier
3) Bridge Rectifier

HALF-WAVE RECTIFIER:

A Half – wave rectifier as shown in **fig 2** is one, which converts a.c. voltage into a pulsating voltage using only one half cycle of the applied a.c. voltage.

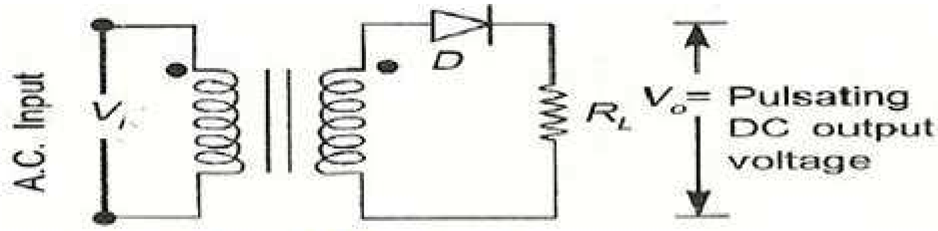


fig 2 Basic structure of Half-Wave Rectifier

The a.c. voltage is applied to the rectifier circuit using step-down transformer-rectifying element i.e., p-n junction diode and the source of a.c. voltage, all connected is series. The a.c. voltage is applied to the rectifier circuit using step-down transformer

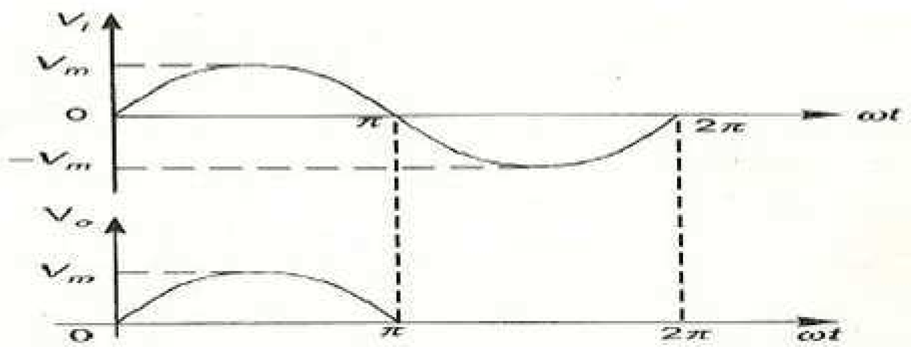


fig 3 Input and output waveforms of a Half wave rectifier

$$V=V_m \sin (wt)$$

The input to the rectifier circuit, Where V_m is the peak value of secondary a.c. voltage.

Operation:

For the positive half-cycle of input a.c. voltage, the diode D is forward biased and hence it conducts. Now a current flows in the circuit and there is a voltage drop across RL. The waveform of the diode current (or) load current is shown in **fig 3**.

For the negative half-cycle of input, the diode D is reverse biased and hence it does not Conduct. Now no current flows in the circuit i.e., i=0 and Vo=0. Thus for the negative half-cycle no power is delivered to the load.

Analysis:

In the analysis of a HWR, the following parameters are to be analyzed.

1. DC output current

2. DC Output voltage
3. R.M.S. Current
4. R.M.S. voltage
5. Rectifier Efficiency (η)
6. Ripple factor (γ)
7. Peak Factor
8. % Regulation
9. Transformer Utilization Factor (TUF)
10. form factor
11. o/p frequency

Let a sinusoidal voltage Vi be applied to the input of the rectifier.
Then $V=V_m \sin(wt)$ Where V_m is the maximum value of the secondary voltage. Let the diode be idealized to piece-wise linear approximation with resistance Rf in the forward direction i.e., in the ON state and Rr (=∞) in the reverse direction i.e., in the OFF state. Now the current 'i' in the diode (or) in the load resistance RL is given by $V=V_m \sin(wt)$

i) AVERAGE VOLTAGE

$$V_{dc} = \frac{1}{T}\int_0^T V d(wt)$$

$$V_{dc} = \frac{1}{T}\int_0^{2\Pi} V(\alpha) d\alpha$$

$$V_{dc} = \frac{1}{2\Pi}\int_{\Pi}^{2\Pi} V(\alpha) d\alpha$$

$$V_{dc} = \frac{1}{2\Pi}\int_0 V_m \sin(wt)$$

$$V_{dc} = \frac{V_m}{\Pi}$$

ii).AVERAGE CURRENT:

$$I_{dc} = \frac{I_m}{\Pi}$$

iii) RMS VOLTAGE:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2 d(wt)}$$

$$V_{rms} = \sqrt{\frac{1}{2\Pi}\int_0^{2\Pi} (V_m sim(wt))^2 d(wt)}$$

$$V_{rms} = \frac{V_m}{2}$$

IV) RMS CURRENT

$$I_{rms} = \frac{I_m}{\Pi}$$

V) PEAK FACTOR

Peak factor = $\frac{peakvalue}{rmsvalue}$

Peak Factor = $\frac{V_m}{(V_m / 2)}$

Peak Factor =2

vi) FORM FACTOR

Form factor= $\frac{Rmsvalue}{averagevalue}$

Form factor= $\frac{(V_m / 2)}{V_m / \Pi}$

Form Factor =1.57

vii) Ripple Factor:

$$\Gamma = \frac{V_{ac}}{V_{dc}}$$

$$V_{ac} = \sqrt{V_{rms}^2 - V_{dc}^2}$$

$$\Gamma = \frac{\sqrt{V_{rms}^2 - V_{dc}^2}}{V_{ac}}$$

$$\Gamma = \sqrt{\frac{V_{rms}^2}{V_{dc}^2} - 1}$$

$$\Gamma = 1.21$$

viii) Efficiency (η):

$$\eta = \frac{o/ppower}{i/ppower} * 100$$

$$\eta = \frac{p_{ac}}{P_{dc}} * 100$$

$$\eta = 40.8$$

ix) Transformer Utilization Factor (TUF):

The d.c. power to be delivered to the load in a rectifier circuit decides the rating of the transformer used in the circuit. Therefore, transformer utilization factor is defined as

$$TUF = \frac{p_{dc}}{P_{ac(rated)}}$$

$$TUF = 0.286.$$

The value of TUF is low which shows that in half-wave circuit, the transformer is not fully utilized.

If the transformer rating is 1 KVA (1000VA) then the half-wave rectifier can deliver

1000 X 0.287 = 287 watts to resistance load.

x) Peak Inverse Voltage (PIV):

It is defined as the maximum reverse voltage that a diode can withstand without destroying the junction. The peak inverse voltage across a diode is the peak of the negative half- cycle. For half-wave rectifier, PIV is Vm.

DISADVANTAGES OF HALF-WAVE RECTIFIER:

1. The ripple factor is high.

2. The efficiency is low.
3. The Transformer Utilization factor is low.

Because of all these disadvantages, the half-wave rectifier circuit is normally not used as a power rectifier circuit.

2.2.2) FULL WAVE RECTIFIER:

A full-wave rectifier converts an ac voltage into a pulsating dc voltage using both half cycles of the applied ac voltage. In order to rectify both the half cycles of ac input, two diodes are used in this circuit. The diodes feed a common load RL with the help of a center-tap transformer. A center-tap transformer is the one, which produces two sinusoidal waveforms of same magnitude and frequency but out of phase with respect to the ground in the secondary winding of the transformer. The full wave rectifier is shown in the **fig 4** below

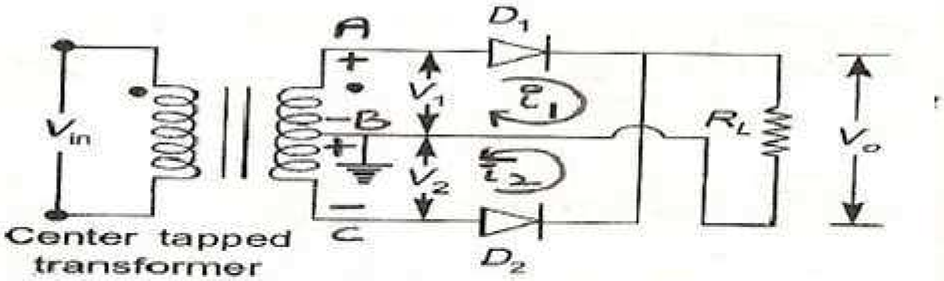


fig 4 Full-Wave Rectifier

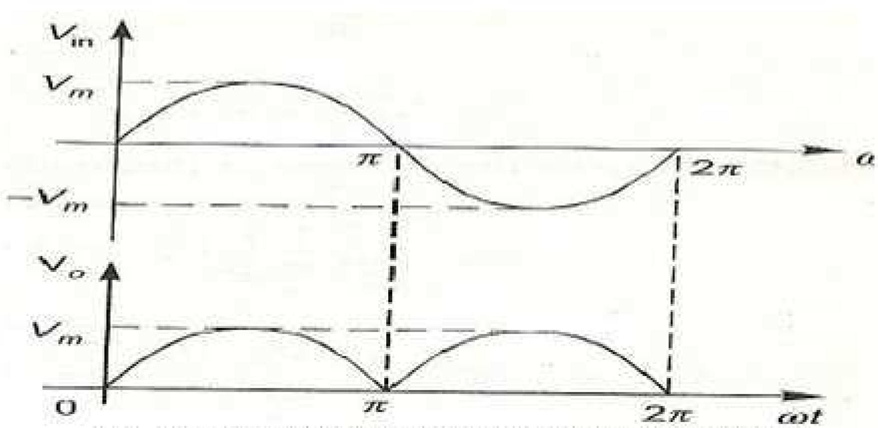


Fig. 5 input and output waveforms of Fullwave rectifier

Fig. 5 shows the input and output wave forms of the ckt.

During positive half of the input signal, anode of diode D1 becomes positive and at the same time the anode of diode D2 becomes negative. Hence D1 conducts and D2 does not conduct. The load current flows through D1 and the voltage drop across RL will be equal to the input voltage.

During the negative half cycle of the input, the anode of D1 becomes negative and the anode of D2 becomes positive. Hence, D1 does not conduct and D2 conducts. The load current flows through D2 and the voltage drop across RL will be equal to the input voltage. It is noted that the load current flows in the both the half cycles of ac voltage and in the same direction through the load resistance.

AVERAGEVOLTAGE

$$V_{dc} = I_{dc}.R_L. = \frac{2\,I_m}{\pi}.R_L \quad \text{We know } I_m = \frac{V_m}{R_S + R_f + R_L}$$

$$\therefore V_{dc} = \frac{2.V_m R_L}{\pi(R_S + R_f + R_L}$$

$$If\,(R_s + R_f) << R_L$$

$$V_{dc} = \frac{2V_m}{\pi} = 0.637V_m.$$

i) **AVERAGE CURRENT**

$$I_{dc} \quad \frac{1}{2\pi}\int_0^{2\pi} i d\theta = \frac{1}{2\pi}\int_0^{2\pi} I_m \sin\theta dt$$

$$= \frac{I_m}{2\pi}\left[\int_0^{\pi} \sin\theta d\theta - \int_{\pi}^{2\pi} \sin\theta d\theta\right]$$

$$= \frac{I_m}{2\pi}[(-2)(-2)]$$

$$= \frac{I_m}{2\pi}.4 = \frac{2\,I_m}{\pi} = 0.637\,I_m.$$

$$\boxed{I_{dc} = 0.637 I_m.}$$

$$\therefore I_{DC}\ FWR = 2\ I_{DC}\ HWR.$$

iii) RMS VOLTAGE:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2 d(wt)}$$

$$V_{rms} = \sqrt{\frac{1}{2\Pi}\int_0^{2\Pi} (V_m sim(wt))^2 d(wt)}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

IV) RMS CURRENT

$$I_{rms} = \frac{2I_m}{\Pi}$$

V) PEAK FACTOR

Peak factor $= \frac{peakvalue}{rmsvalue}$

Peak Factor $= \frac{V_m}{(V_m / 2)}$

Peak Factor =2

vi) FORM FACTOR

Form factor$= \frac{Rms\ value}{average\ value}$

Form factor$= \frac{(V_m / \sqrt{2})}{2V_m / \Pi}$

Form Factor =1.11

vii) Ripple Factor:

$$\gamma = \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1}$$

for FWR,

$$I_{rms} = \frac{I_m}{\sqrt{2}} \quad \& \quad I_{DC} = \frac{2I_m}{\pi}$$

$$\therefore \gamma_{FWR} = \sqrt{\left(\frac{I_m}{\sqrt{2}} / \frac{2I_m}{\pi}\right)^2 - 1}$$

$$= \sqrt{\left(\frac{\pi}{2\sqrt{2}}\right)^2 - 1}$$

$$= \sqrt{\left(\frac{3.1416}{2\times 1.414}\right)^2 - 1} = 0.483$$

viii) Efficiency (η):

$$\eta = \frac{o/ppower}{i/ppower} * 100$$

$$\eta = \frac{p_{dc}}{p_{ac}} \times 100\%$$

For FWR, $p_{dc} = I_{dc}^2 . R_L = \left(\frac{2}{\pi} . I_m\right)^2 . R_L$

$P_{ac} = I^2_{rms} (R_f + R_S + R_L)$

$$\left(\frac{I_m}{\sqrt{2}}\right)^2 (R_f + R_S + R_L)$$

$$\eta = \frac{\frac{I^2{}_m 4}{\pi^2} . R_L}{\frac{I^2 m^2}{2} . (R_f + R_s + R_L)}$$

$$If (R_f + R_S) << R_L$$

$$\eta = \frac{4}{\pi^2} . \frac{2}{1} = \frac{8}{\pi^2} = 0.812 = 81.2\%$$

ix) Transformer Utilization Factor (TUF):

The d.c. power to be delivered to the load in a rectifier circuit decides the rating of the transformer used in the circuit. So, transformer utilization factor is defined as

$$TUF = \frac{p_{dc}}{P_{ac(rated)}}$$

a) TUF (Secondary) = $\dfrac{P_{dc} \textit{ delivered to load}}{\textit{AC power rating of transformer secondary}}$

b) Since both the windings are used TUF $_{FWR}$ = 2 TUF $_{HWR}$

= 2 x 0.287 = 0.574

c) TUF primary = Rated efficiency = $\dfrac{P_{dc}}{P_{ac}} \times 100$ = 81.2%

d) Average = $\dfrac{0.812 + 0.574}{2} = 0.693$

x) Peak Inverse Voltage (PIV):

It is defined as the maximum reverse voltage that a diode can withstand without destroying the junction. The peak inverse voltage across a diode is the peak of the negative half- cycle. For half-wave rectifier, PIV is 2Vm

xi) % Regulation

$$\text{Voltage regulation} = \frac{I_{dc}(R_s + R_f)}{\frac{2V_m}{\pi} - I_{DC}(R_f + R_S)}$$

Advantages

1) Ripple factor = 0.482 (against 1.21 for HWR)
2) Rectification efficiency is 0.812 (against 0.405 for HWR)
3) Better TUF (secondary) is 0.574 (0.287 for HWR)
4) No core saturation problem

Disadvantages:

1) Requires center tapped transformer.

2.2.3) BRIDGE RECTIFIER

Another type of circuit that produces the same output waveform as the full wave rectifier circuit above, is that of the **Full Wave Bridge Rectifier**. This type of single phase rectifier uses four individual rectifying diodes connected in a closed loop "bridge" configuration to produce the desired output. The main advantage of this bridge circuit is that it does not require a special centre tapped transformer, thereby reducing its size and cost. The single secondary winding is connected to one side of the diode bridge network and the load to the other side as shown below.

The Diode Bridge Rectifier

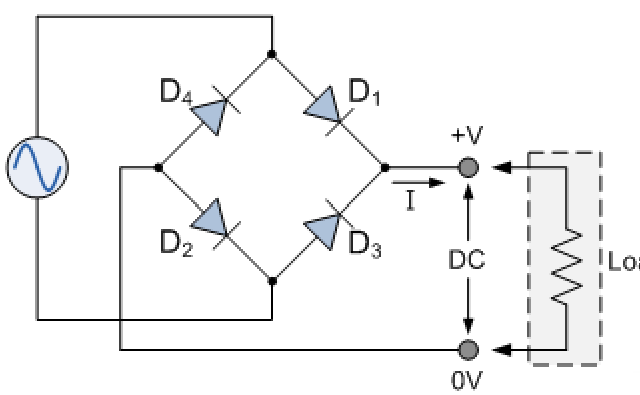


fig 6

The four diodes labeled D_1 to D_4 are arranged in "series pairs" with only two diodes conducting current during each half cycle. During the positive half cycle of the supply, diodes D1 and D2 conduct in series while diodes D3 and D4 are reverse biased and the current flows through the load as shown below (fig 7).

The Positive Half-cycle

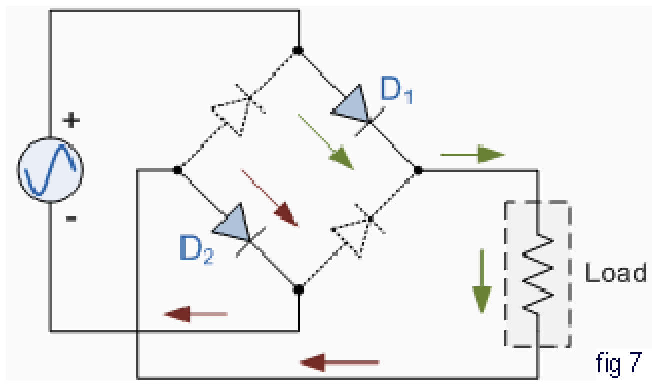


fig 7

The Negative Half-cycle

During the negative half cycle of the supply, diodes D3 and D4 conduct in series (fig 8), but diodes D1 and D2 switch "OFF" as they are now reverse biased. The current flowing through the load is the same direction as before.

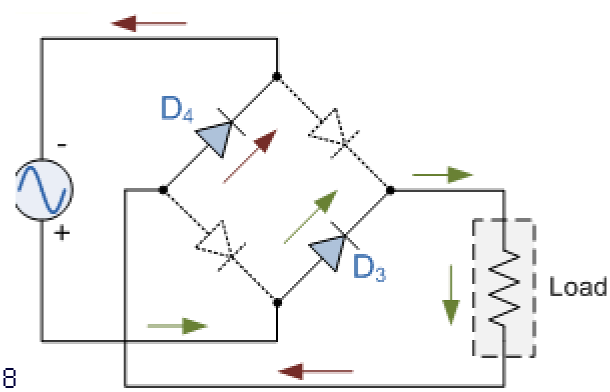


fig 8

As the current flowing through the load is unidirectional, so the voltage developed across the load is also unidirectional the same as for the previous two diode full-wave rectifier, therefore the average DC voltage across the load is $0.637V_{max}$. However in reality, during each half cycle the current flows through two diodes instead of just one so the amplitude of the output voltage is two voltage drops (2 x 0.7 = 1.4V) less than the input V_{MAX} amplitude. The ripple frequency is now twice the supply frequency (e.g. 100Hz for a 50Hz supply)

Therefore, the following expressions are same as that of full wave rectifier.

a) Average current $I_{dc} = \frac{2I_m}{\pi}$

b) RMS current $I_{rms} = \frac{I_m}{\sqrt{2}}$

c) DC output voltage (no.load) V_{DC} $\frac{2V_m}{\pi}$

d) Ripple factor $\gamma = 0.482$

e) Rectification efficiency = $\eta = 0.812$

f) DC output voltage full load.

$= \quad V_{DCFL} = \frac{2V_m}{\pi} - I_{dc}(R_S + 2R_f)$; i.e., less by one diode loss.

TUF of both primary & secondary are 0.812 therefore TUF overall is 0.812 (better than FWR with 0.693)

Comparison:

Sl. No.	Parameter	HWR	FWR	BR
1	No. of diodes	1	2	4
2	PIV of diodes	V_m	$2\ V_m$	V_m
3	Secondary voltage (rms)	V	V-0-V	V
4	DC output voltage at no load	$\frac{V_m}{\pi} = 0.318\ V_m$	$\frac{2V_m}{\pi} = 0.636\ V_m$	$\frac{2V_m}{\pi} = 0.636\ V_m$
5	Ripple factor γ	1.21	0.482	0.482
6	Ripple frequency	f	2f	2f
7	Rectification efficiency η	0.406	0.812	0.812
8	TUF	0.287	0.693	0.812

BIPOLAR JUNCTION TRANSISTOR

3.1 INTRODUCTION

A bipolar junction transistor (BJT) is a three terminal device in which operation depends on the interaction of both majority and minority carriers and hence the name bipolar. The BJT is analogus to vacuum triode and is comparatively smaller in size. It is used as amplifier and oscillator circuits, and as a switch in digital circuits. It has wide applications in computers, satellites and other modern communication systems.

3.2 CONSTRUCTION OF BJT AND ITS SYMBOLS

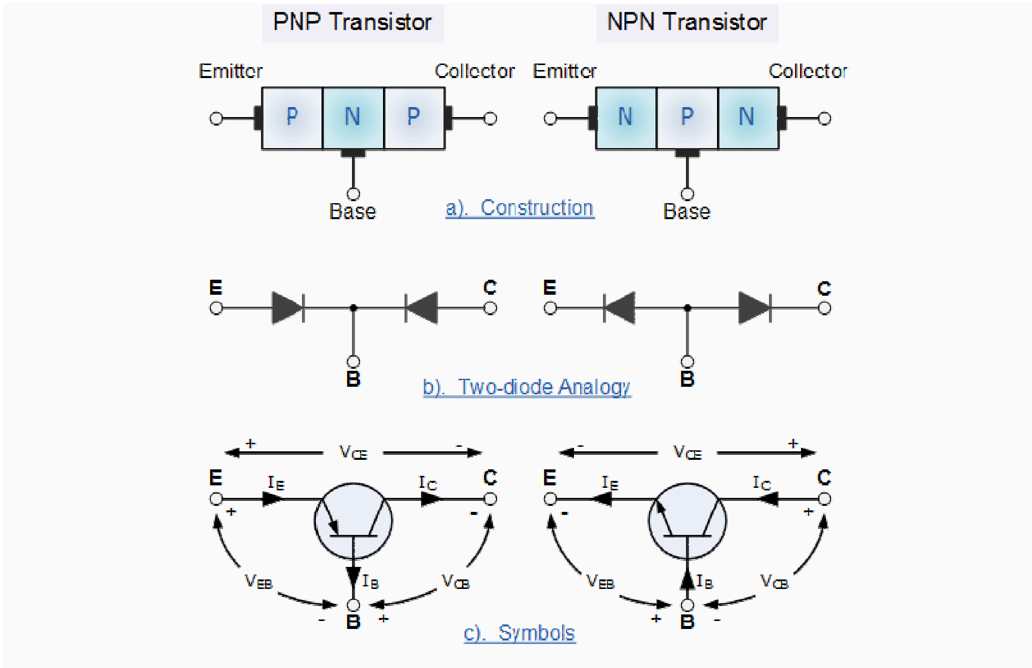
The **Bipolar Transistor** basic construction consists of two PN-junctions producing three connecting terminals with each terminal being given a name to identify it from the other two. These three terminals are known and labeled as the Emitter (E), the Base (B) and the Collector (C) respectively. There are two basic types of bipolar transistor construction, PNP and NPN, which basically describes the physical arrangement of the P-type and N-type semiconductor materials from which they are made.

Transistors are three terminal active devices made from different semiconductor materials that can act as either an insulator or a conductor by the application of a small signal voltage. The transistor's ability to change between these two states enables it to have two basic functions: "switching" (digital electronics) or "amplification" (analogue electronics). Then bipolar transistors have the ability to operate within three different regions:

- 1. Active Region - the transistor operates as an amplifier and $I_c = \beta.I_b$
- 2. Saturation - the transistor is "fully-ON" operating as a switch and $I_c = I(saturation)$
- 3. Cut-off - the transistor is "fully-OFF" operating as a switch and $I_c = 0$

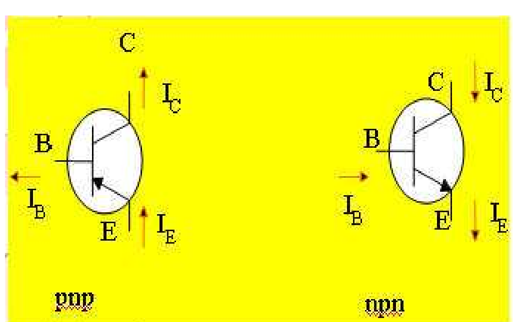
Bipolar Transistors are current regulating devices that control the amount of current flowing through them in proportion to the amount of biasing voltage applied to their base terminal acting like a current-controlled switch. The principle of operation of the two transistor types PNP and NPN, is exactly the same the only difference being in their biasing and the polarity of the power supply for each type.

Bipolar Transistor Construction



The construction and circuit symbols for both the PNP and NPN bipolar transistor are given above with the arrow in the circuit symbol always showing the direction of "conventional current flow" between the base terminal and its emitter terminal. The direction of the arrow always points from the positive P-type region to the negative N-type region for both transistor types, exactly the same as for the standard diode symbol.

TRANSISTOR CURRENT COMPONENTS:



The above fig 2 shows the various current components, which flow across the forward biased emitter junction and reverse- biased collector junction. The emitter current I_E consists of hole current I_{PE} (holes crossing from emitter into base) and electron current I_{nE} (electrons crossing from base into emitter).The ratio of hole to electron currents, I_{pE} / I_{nE} , crossing the emitter junction is proportional to the ratio of the conductivity of the p material to that of the n material. In a transistor, the doping of that of the emitter is made much larger than the doping of the base. This feature ensures (in p-n-p transistor) that the emitter current consists an almost entirely of holes. Such a situation is desired since the current which results from electrons crossing the emitter junction from base to emitter does not contribute carriers, which can reach the collector.

Not all the holes crossing the emitter junction J_E reach the the collector junction J_C

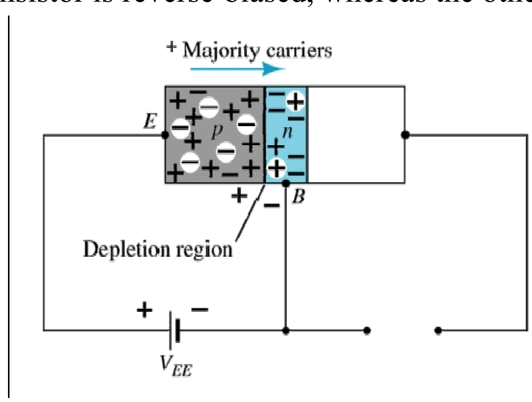
Because some of them combine with the electrons in n-type base. If I_{pC} is hole current at junction J_C there must be a bulk recombination current (I_{PE}- I_{pC}) leaving the base.

Actually, electrons enter the base region through the base lead to supply those charges, which have been lost by recombination with the holes injected in to the base across J_E. If the emitter were open circuited so that I_E=0 then I_{pC} would be zero. Under these circumstances, the base and collector current I_C would equal the reverse saturation current I_{CO}. If $I_E \neq 0$ then

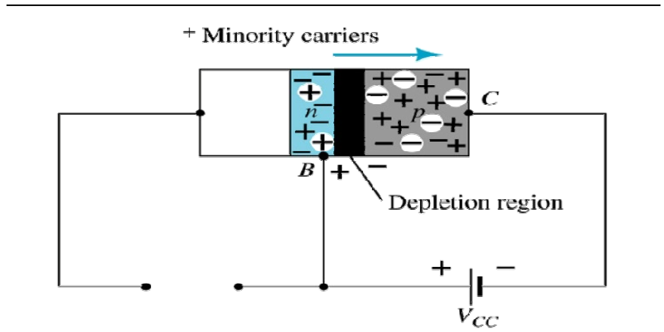
$$I_C = I_{CO} - I_{pC}$$

For a p-n-p transistor, I_{CO} consists of holes moving across J_C from left to right (base to collector) and electrons crossing J_C in opposite direction. Assumed referenced direction for I_{CO} i.e. from right to left, then for a p-n-p transistor, I_{CO} is negative. For an n-p-n transistor, I_{CO} is positive. The basic operation will be described using the pnp transistor. The operation of the pnp transistor is exactly the same if the roles played by the electron and hole are interchanged.

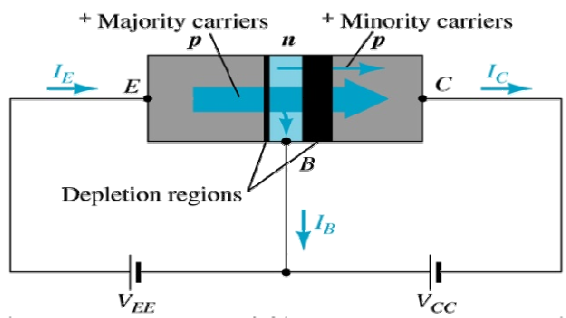
One p-n junction of a transistor is reverse-biased, whereas the other is forward-biased.



Forward-biased junction of a PNP transistor



Reverse-biased junction of a PNP transistor



Both biasing potentials have been applied to a pnp transistor and resulting majority and minority carrier flows indicated.

Majority carriers (+) will diffuse across the forward-biased p-n junction into the n-type material.

A very small number of carriers (+) will through n-type material to the base terminal. Resulting IB is typically in order of microamperes.

The large number of majority carriers will diffuse across the reverse-biased junction into the p-type material connected to the collector terminal

Applying KCL to the transistor:

$$I_E = I_C + I_B$$

The comprises of two components – the majority and minority carriers

$$I_C = I_{C\,majority} + I_{CO\,minority}$$

I_{CO} – I_C current with emitter terminal open and is called leakage current

Various parameters which relate the current components is given below

Emitter efficiency:

$$\gamma = \frac{currentofinjectedcarriersatJ_E}{totalemittercurrent}$$

$$\gamma = \frac{I_{PE}}{I_{pE} + I_{nE}} = \frac{I_{pE}}{I_{nE}}$$

Transport Factor:

$$\beta^* = \frac{injectedcarriercurrentreachingJ_C}{injectedcarrierncurrentatJ_E}$$

$$\beta^* = \frac{I_{pC}}{I_{nE}}$$

Large signal current gain:

The ratio of the negative of collector current increment to the emitter current change from zero (cut-off)to I_E the large signal current gain of a common base transistor.

$$\alpha = \frac{-(I_C - I_{CO})}{I_E}$$

Since I_C and I_E have opposite signs, then α, as defined, is always positive. Typically numerical values of α lies in the range of 0.90 to 0.995

$$\alpha = \frac{I_{pC}}{I_E} = \frac{I_{pC}}{I_{nE}} * \frac{I_{pE}}{I_E} \quad \alpha = \beta * \gamma$$

The transistor alpha is the product of the transport factor and the emitter efficiency. This statement assumes that the collector multiplication ratio α^* is unity. α^* is the ratio of total current crossing J_C to hole arriving at the junction.

Bipolar Transistor Configurations

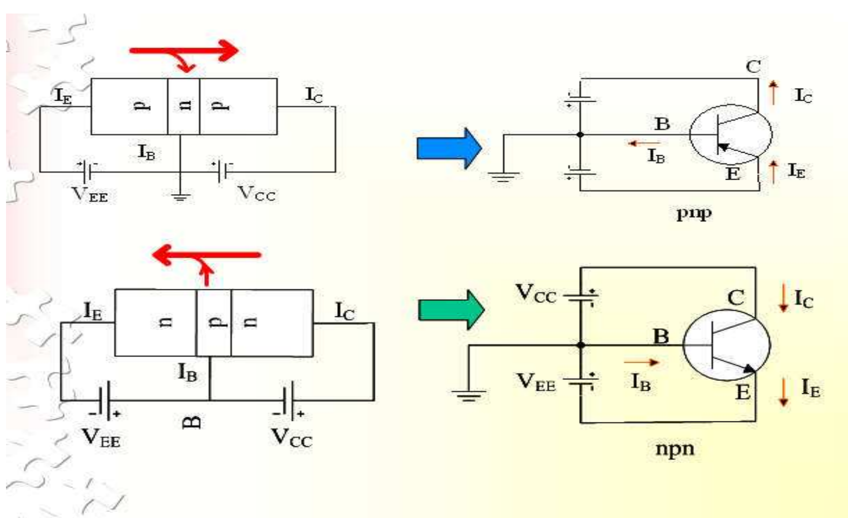
As the **Bipolar Transistor** is a three terminal device, there are basically three possible ways to connect it within an electronic circuit with one terminal being common to both the input and output. Each method of connection responding differently to its input signal within a circuit as the static characteristics of the transistor vary with each circuit arrangement.

- 1. Common Base Configuration - has Voltage Gain but no Current Gain.
- 2 Common Emitter Configuration - has both Current and Voltage Gain.
- 3. Common Collector Configuration - has Current Gain but no Voltage Gain.

3.5 COMMON-BASE CONFIGURATION

Common-base terminology is derived from the fact that the: base is common to both input and output of t configuration. base is usually the terminal closest to or at ground potential. Majority carriers can cross the reverse-biased junction because the injected majority carriers will appear as minority carriers in the n-type material. All current directions will refer to conventional (hole) flow and the arrows in all electronic symbols have a direction defined by this convention.

Note that the applied biasing (voltage sources) are such as to establish current in the direction indicated for each branch.

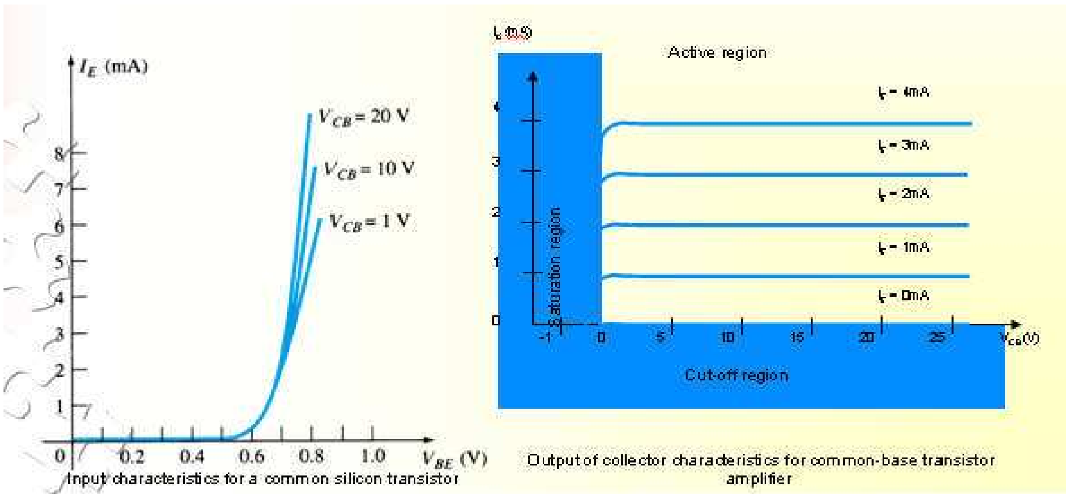


To describe the behavior of common-base amplifiers requires two set of characteristics:

1. Input or driving point characteristics.
2. Output or collector characteristics

The output characteristics have 3 basic regions:

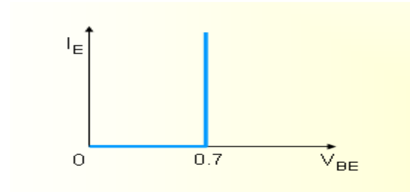
- Active region –defined by the biasing arrangements
- Cutoff region – region where the collector current is 0A
- Saturation region- region of the characteristics to the left of V_{CB} = 0V



Active region	Saturation region	Cut-off region
•IE increased, Ic increased •BE junction forward bias and CB junction reverse bias •Refer to the graf, Ic ≈ IE •Ic not depends on VCB •Suitable region for the transistor working as amplifier	• BE and CB junction is forward bias • Small changes in VCB will cause big different to Ic • The allocation for this region is to the left of VCB= 0 V.	• Region below the line of IE=0 A • BE and CB is reverse bias • no current flow at collector, only leakage current

The curves (output characteristics) clearly indicate that a first approximation to the relationship between IE and IC in the active region is given by

$I_C \approx IE$ Once a transistor is in the 'on' state, the base-emitter voltage will be assumed to be V_{BE} = 0.7V



In the dc mode the level of I_C and I_E due to the majority carriers are related by a quantity called alpha $\alpha = \alpha_{dc}$

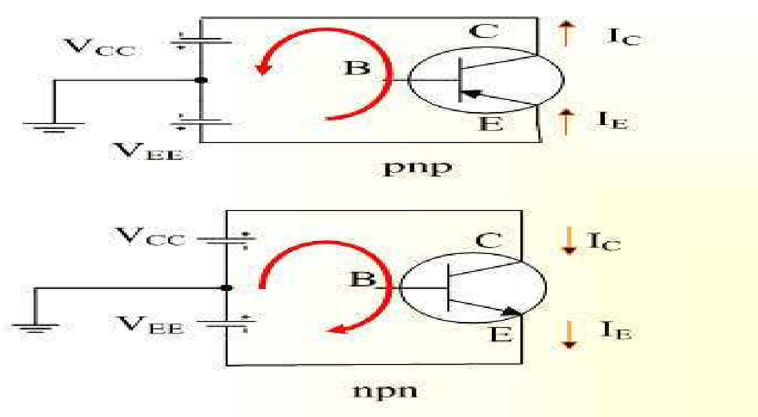
$$I_C = \alpha I_E + I_{CBO}$$

It can then be summarize to $I_C = \alpha I_E$ (ignore I_{CBO} due to small value)

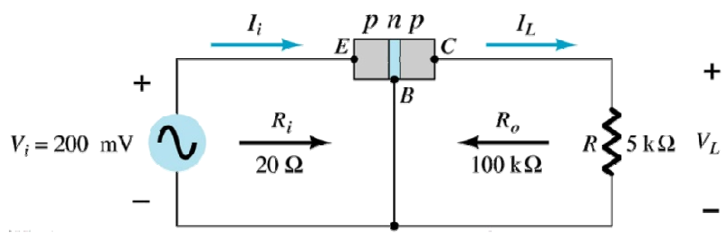
For ac situations where the point of operation moves on the characteristics curve, an ac alpha defined by α_{ac}

Alpha a common base current gain factor that shows the efficiency by calculating the current percent from current flow from emitter to collector. The value of α is typical from 0.9 ~ 0.998.

Biasing: Proper biasing CB configuration in active region by approximation $I_C \approx I_E$ ($I_B \approx 0$ uA)



TRANSISTOR AS AN AMPLIFIER



Common-Emitter Configuration

It is called common-emitter configuration since : emitter is common or reference to both input and output terminals.emitter is usually the terminal closest to or at ground potential.

Almost amplifier design is using connection of CE due to the high gain for current and voltage.

Two set of characteristics are necessary to describe the behavior for CE ;input (base terminal) and output (collector terminal) parameters.

Proper Biasing common-emitter configuration in active region

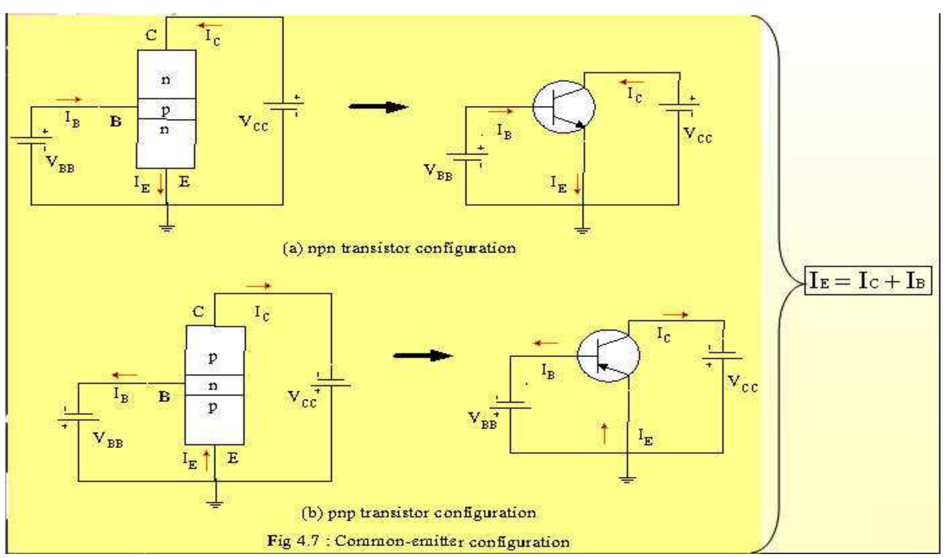


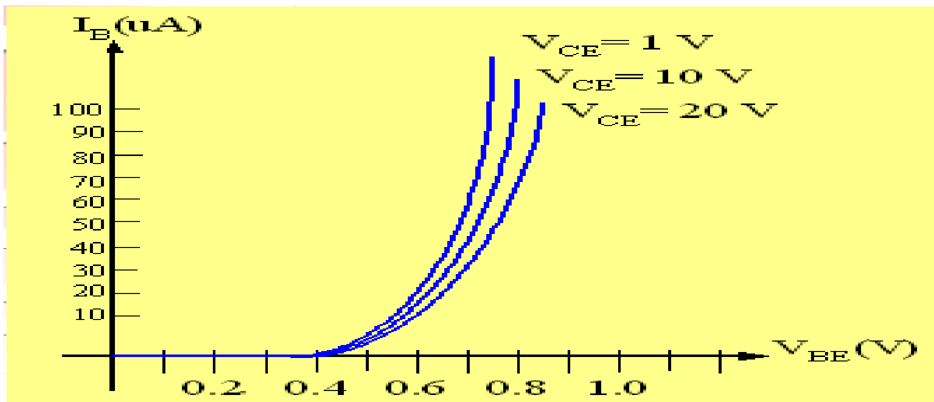
Fig 4.7 : Common-emitter configuration

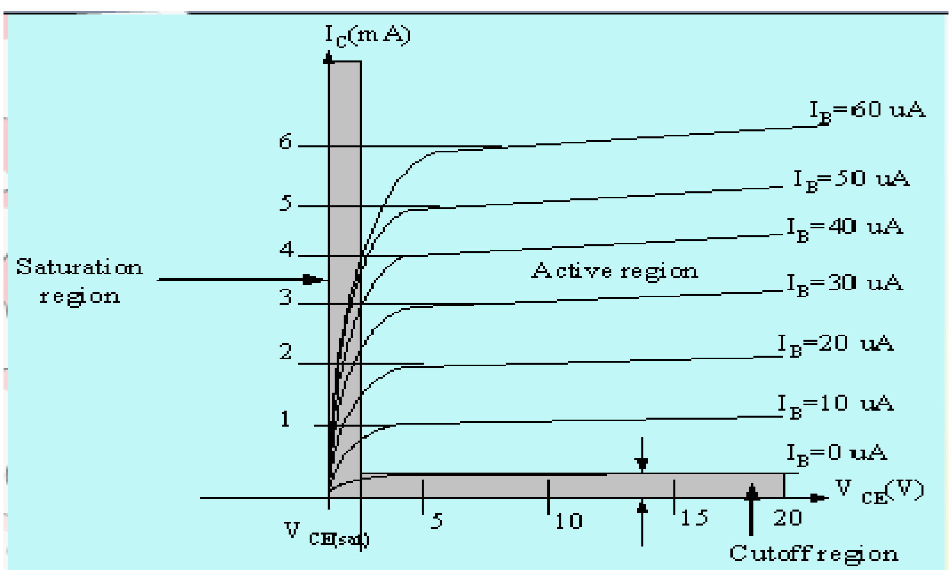
I_B is microamperes compared to miliamperes of I_C.

I_B will flow when V_{BE} > 0.7V for silicon and 0.3V for germanium

Before this value I_B is very small and no I_B.

Base-emitter junction is forward bias Increasing V_{CE} will reduce I_B for different values.





Output characteristics for acommon-emitter npn transistor

For small V_{CE} ($V_{CE} < V_{CESAT}$, I_C increase linearly with increasing of V_{CE}

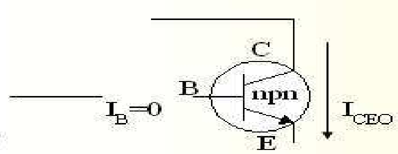
$V_{CE} > V_{CESAT}$ I_C not totally depends on V_{CE} → constant I_C

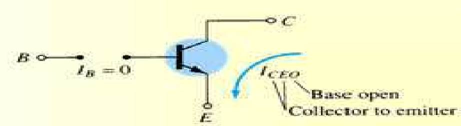
I_B(uA) is very small compare to I_C (mA). Small increase in I_B cause big increase in I_C

I_B=0 A → I_{CEO} occur.

Noticing the value when I_C=0A. There is still some value of current flows.

Active region	Saturation region	Cut-off region
• B-E junction is forward bias • C-B junction is reverse bias • can be employed for voltage, current and power amplification	• B-E and C-B junction is forward bias, thus the values of I_B and I_C is too big. • The value of V_{CE} is so small. • Suitable region when the transistor as a logic switch. • NOT and avoid this region when the transistor as an amplifier.	• region below I_B=0μA is to be avoided if an undistorted o/p signal is required • B-E junction and C-B junction is reverse bias • I_B=0, I_C not zero, during this condition $I_C=I_{CEO}$ where is this current flow when B-E is reverse bias.



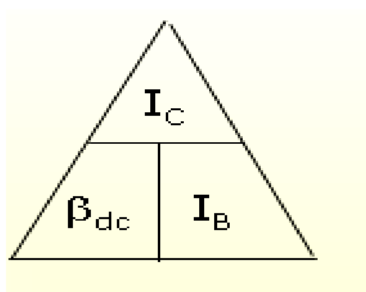


Beta (β) or amplification factor

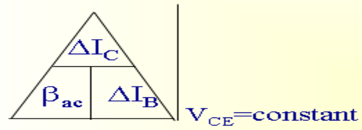
The ratio of dc collector current (IC) to the dc base current (IB) is dc beta (βdc) which is dc current gain where IC and IB are determined at a particular operating point, Q-point (quiescent point). It's define by the following equation:

$$30 < \beta dc < 300 \rightarrow 2N3904$$

On data sheet, $\beta_{dc}=hfe$ with *h* is derived from ac hybrid equivalent cct. FE are derived from forward-current amplification and common-emitter configuration respectively.



For ac conditions, an ac beta has been defined as the changes of collector current (I_C) compared to the changes of base current (I_B) where I_C and I_B are determined at operating point. On data sheet, β_{ac}=*hfe* It can defined by the following equation:



From output characteristics of commonemitter configuration, find β_{ac} and β_{dc} with an

Operating point at I_B=25 μA and V_{CE} =7.5V

$$\beta_{ac} = \frac{\Delta I_C}{\Delta I_B}\bigg|_{V_{ce} = \text{constant}}$$

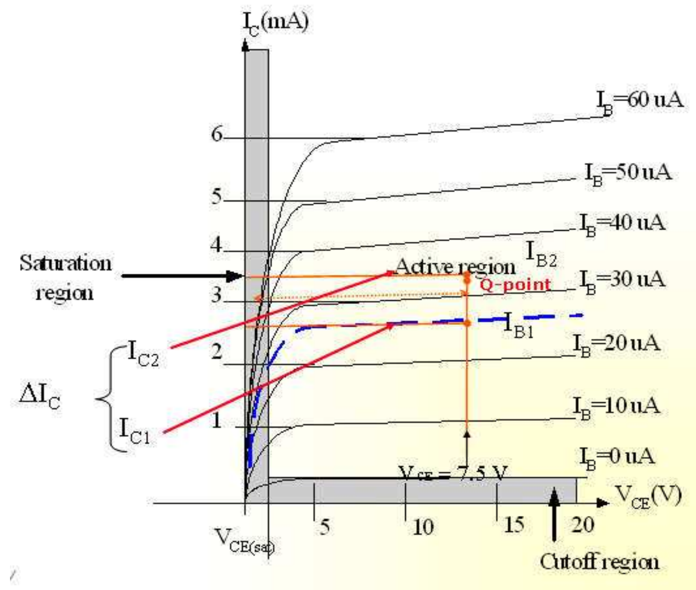
$$= \frac{I_{C2} - I_{C1}}{I_{B2} - I_{B1}} = \frac{3.2m - 2.2m}{30\mu - 20\mu}$$

$$= \frac{1m}{10\mu} = 100$$

$$\beta_{dc} = \frac{I_C}{I_B}$$

$$= \frac{2.7m}{25\mu}$$

$$= 108$$



Relationship analysis between α and β

CASE 1

$$I_E = I_C + I_B \qquad (1)$$

subtitute equ. $I_C = \beta I_B$ into (1) we get

$$I_E = (\beta + 1) I_B$$

CASE 2

known : $\alpha = \frac{I_C}{I_E} \Rightarrow I_E = \frac{I_C}{\alpha} \qquad (2)$

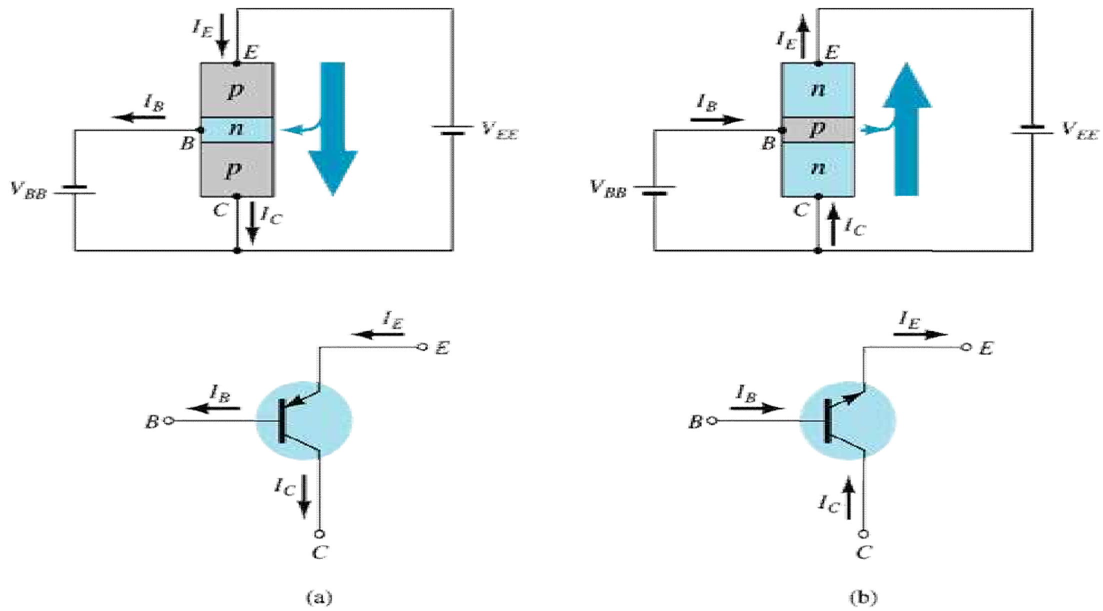
known : $\beta = \frac{I_C}{I_B} \Rightarrow I_B = \frac{I_C}{\beta} \qquad (3)$

subtitute (2) and (3) into (1) we get,

$$\alpha = \frac{\beta}{\beta + 1} \quad \text{and} \quad \beta = \frac{\alpha}{1 - \alpha}$$

3.7 COMMON – COLLECTOR CONFIGURATION

Also called emitter-follower (EF). It is called common-emitter configuration since both the signal source and the load share the collector terminal as a common connection point.The output voltage is obtained at emitter terminal. The input characteristic of common-collector configuration is similar with common-emitter. configuration.Common-collector circuit configuration is provided with the load resistor connected from emitter to ground.It is used primarily for impedance-matching purpose since it has high input impedance and low output impedance.



For the common-collector configuration, the output characteristics are a plot of I_E vs V_{CE} for a range of values of I_B.

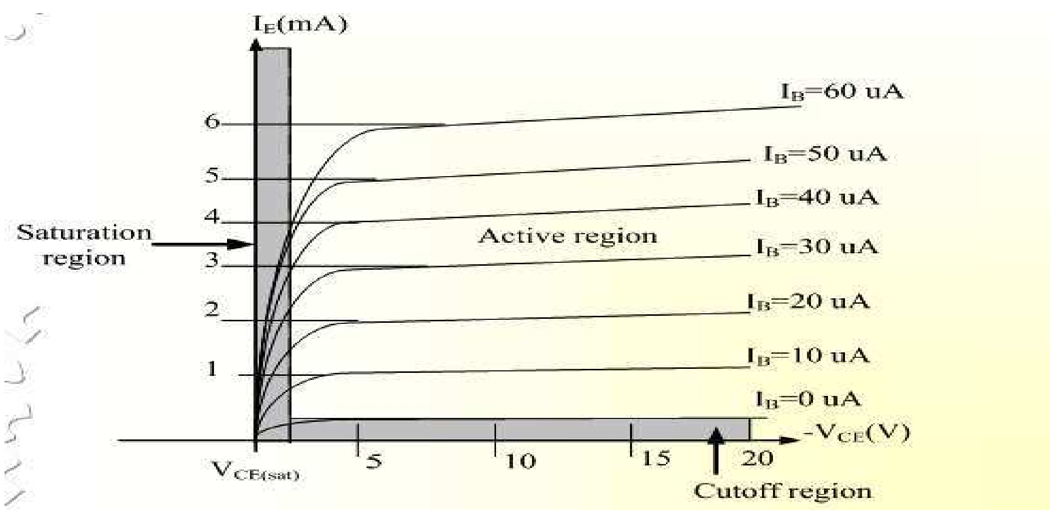


Fig 4.9 : Output characteristic in CC configuration for npn transistor

Limits of operation

Many BJT transistor used as an amplifier. Thus it is important to notice the limits of operations. At least 3 maximum values are mentioned in data sheet.

There are:

a) Maximum power dissipation at collector: P_{Cmax} or P_D

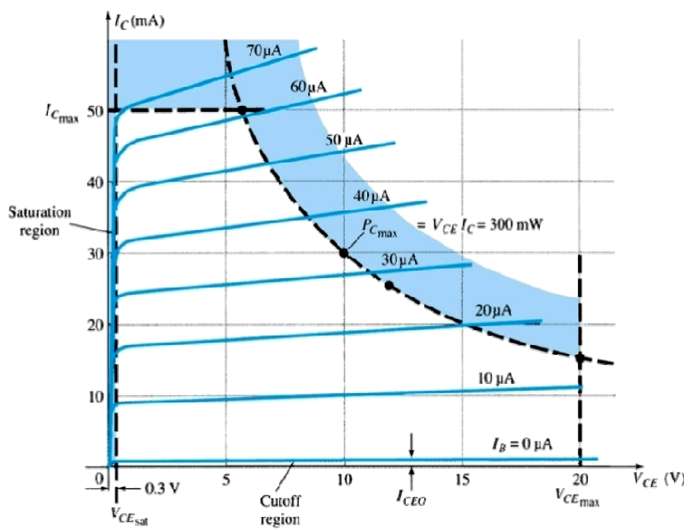
b) Maximum collector-emitter voltage: V_{CEmax} sometimes named as $V_{BR(CEO)}$ or V_{CEO}.

c) Maximum collector current: ICmax

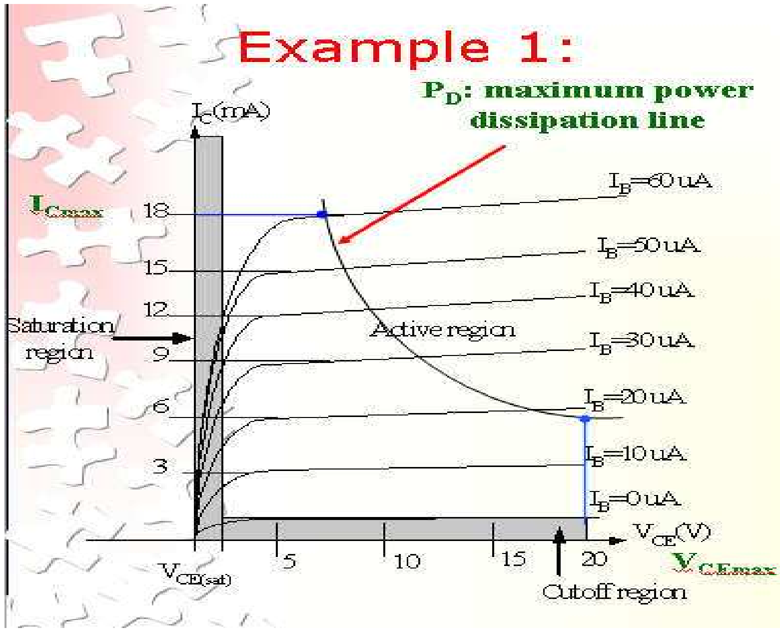
There are few rules that need to be followed for BJT transistor used as an amplifier. The rules are: transistors need to be operating in active region!

$I_C < I_{Cmax}$

$P_C < P_{Cmax}$



Note: V_{CE} is at maximum and I_C is at minimum ($I_{CMAX}=I_{CEO}$) in the cutoff region. I_C is at maximum and V_{CE} is at minimum (V_{CE} max = V_{cesat} = V_{CEO}) in the saturation region. The transistor operates in the active region between saturation and cutoff.

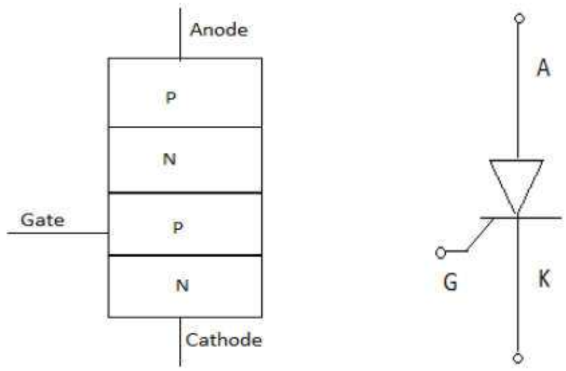


Silicon Controlled Rectifier (SCR)

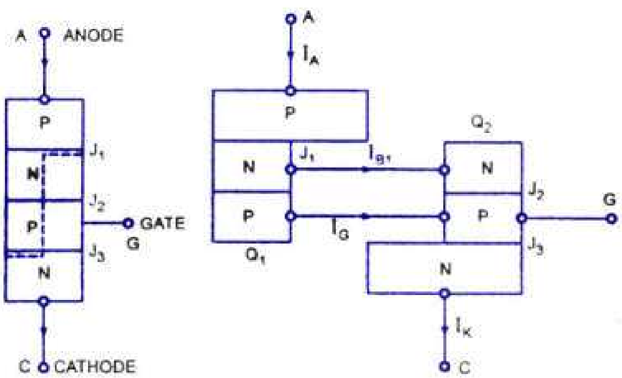
Three terminals
anode - P-layer
cathode - N-layer (opposite end)
gate - P-layer near the cathode
Three junctions - four layers
Connect power such that the anode is positive with respect to the cathode - no current will flow. A silicon controlled rectifier is a semiconductor device that acts as a true electronic switch. it can change alternating current and at the same time can control the amount of power fed to the load. SCR combines the features of a rectifier and a transistor.

CONSTRUCTION

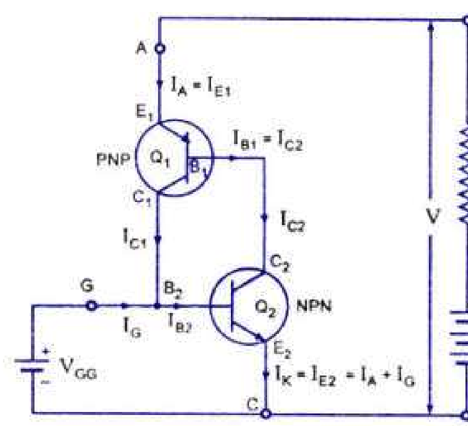
When a pn junction is added to a junction transistor the resulting three pn junction device is called a SCR. ordinary rectifier (pn) and a junction transistor (npn) combined in one unit to form pnpn device. three terminals are taken : one from the outer p- type material called anode a second from the outer n- type material called cathode K and the third from the base of transistor called Gate. GSCR is a solid state equivalent of thyratron. the gate anode and cathode of SCR correspond to the grid plate and cathode of thyratron SCR is called thyristor



Two Transistor analogy of SCR



SCR Split-up into Two Transistors

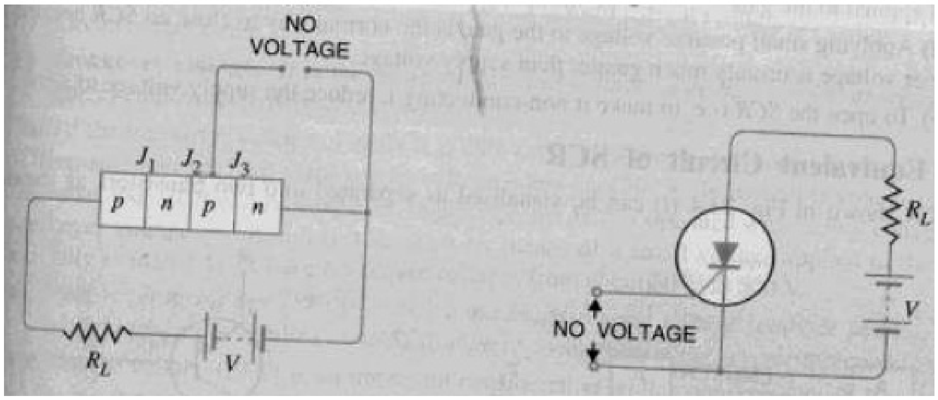


Two Transistor Equivalent Circuit of An SCR

WORKING PRINCIPLE

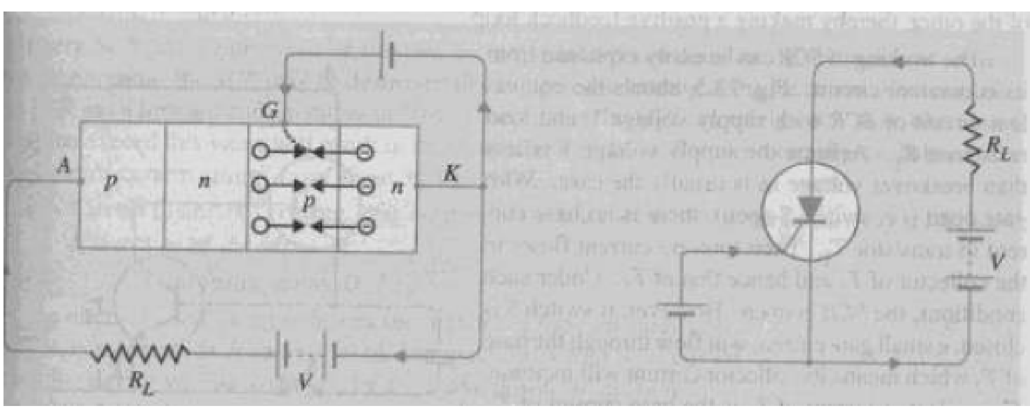
Load is connected in series with anode the anode is always kept at positive potential w.r.t cathode.

WHEN GATE IS OPEN



No voltage applied to the gate, j2 is reverse biased while j1 and j3 are FB . J1 and J3 is just in npn transistor with base open .no current flows through the load RL and SCR is cut off. if the applied voltage is gradually increased a stage is reached when RB junction J2 breakdown .the SCR now conducts heavily and is said to be ON state. the applied voltage at which SCR conducts heavily without gate voltage is called Break over Voltage.

WHEN GATE IS POSITIVE W.R.T CATHODE



The SCR can be made to conduct heavily at smaller applied voltage by applying small positive potential to the gate.J3 is FB and J2 is RB the electron from n type material start moving across J3 towards left holes from p type toward right. electrons from j3 are attracted across junction J2 and gate current starts flowing. as soon as gate current flows anode current increases. the increased anode current in turn makes more electrons available at J2 breakdown and SCR starts conducting heavily. the gate looses all control if the gate voltage is removed anode current does not decrease at all. The only way to stop conduction is to reduce the applied voltage to zero.

BREAKOVER VOLTAGE

It is the minimum forward voltage gate being open at which SCR starts conducting heavily i.e turned on

PEAK REVERSE VOLTAGE(PRV)

It is the maximum reverse voltage applied to an SCR without conducting in the reverse direction

HOLDING CURRENT

It is the maximum anode current gate being open at which SCR is turned off from on conditions.

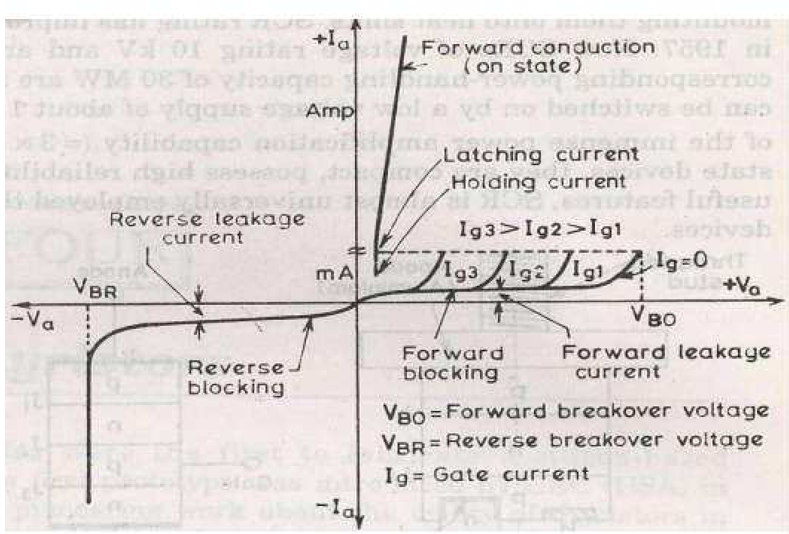
FORWARD CURRENT RATING

It is the maximum anode current that an SCR is capable of passing without destruction

CIRCUIT FUSING RATING

It is the product of square of forward surge current and the time of duration of the surge

VI CHARACTERISTICS OF SCR



FORWARD CHARCTERISTICS

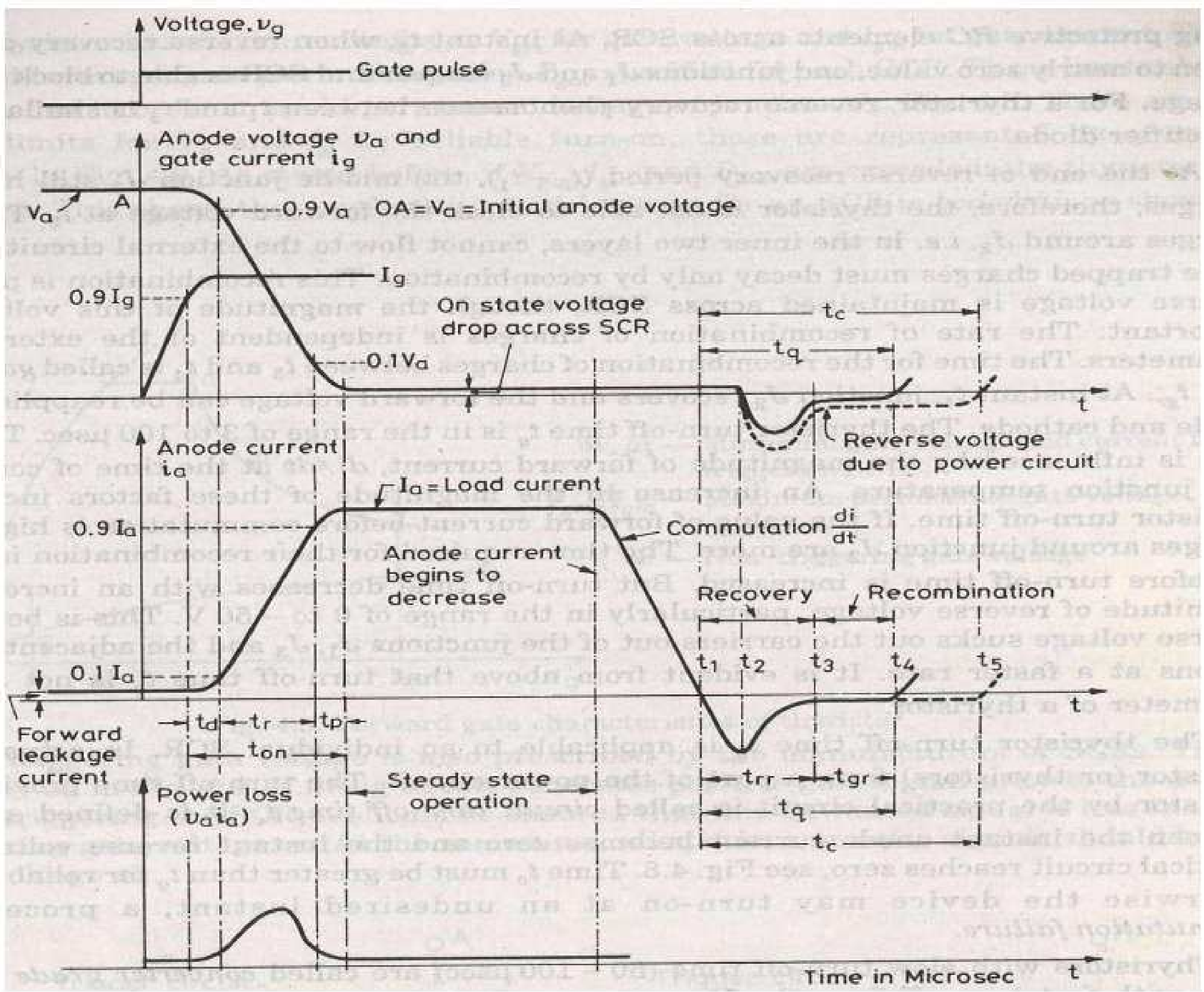
When anode is +ve w.r.t cathode the curve between V & I is called Forward characteristics. OABC is the forward characteristics of the SCR at Ig =0. if the supplied voltage is increased from zero point A is reached .SCR starts conducting voltage across SCR suddenly drops (dotted curve AB) most of supply voltage appears across RL

REVERSE CHARCTERISTICS

When anode is –ve w.r.t.cathode the curve b/w V&I is known as reverse characteristics reverse voltage come across SCR when it is operated with ac supply reverse voltage is increased anode current remains small avalanche breakdown occurs and SCR starts conducting heavily is known as reverse breakdown voltage

SCR as a switch

SCR DYNAMIC CHARACTERISTICS



Applications

SCR as a static contactor
SCR for power control
SCR for speed control of d.c. shunt motor
over light detector

UNIT – V

CATHODE RAY OSCILLOSCOPE

Cathode Ray Oscilloscope (CRO)

The Cathode Ray Oscilloscope is an instrument which we use in laboratory to display measure and analyze various waveforms of various electrical circuit and electronic circuits. Actually cathode ray oscilloscope is very fast X-Y plotters that can display an input signal versus time or other signal. Cathode ray oscilloscope uses luminous spot which is produced by striking the beam of electrons and this luminous spot moves in response variation in the input quantity. At this moment one question must be arise in our mind that why we are using only an electron beam? The reason behind this is low effects of beam of electrons that can be used for following the changes in the instantaneous values of rapidly changing input quantity. The general forms of cathode ray oscilloscope operate on voltages.

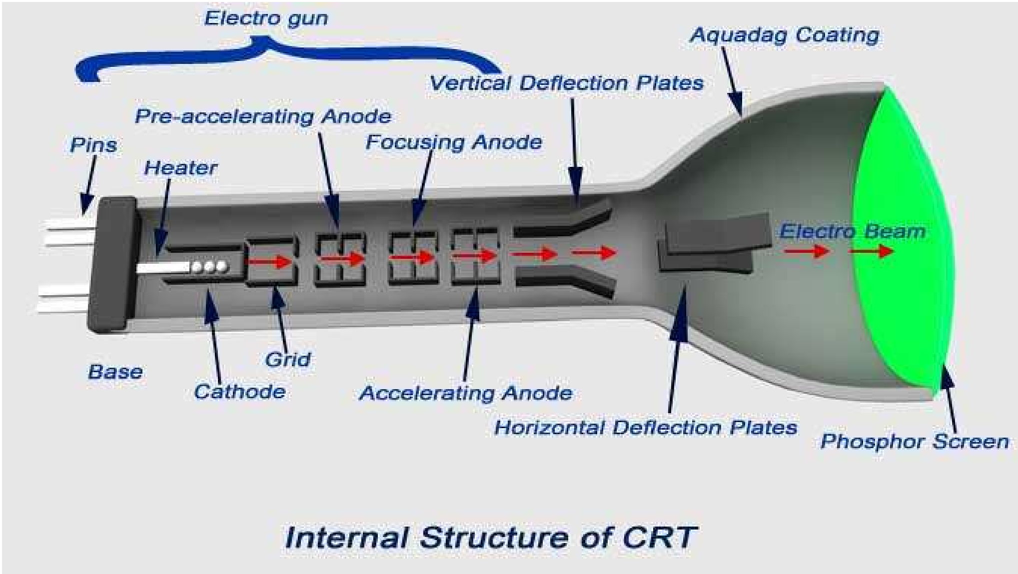
So the input quantity that we have talked above is voltage. Nowadays, with the help of transducers it is possible to convert various physical quantities like current, pressure, acceleration etc to voltage thus it enable us to have a visual representations of these various quantities on cathode ray oscilloscope. Now let us look at the constructional details of the cathode ray oscilloscope.

Construction of Cathode Ray Oscilloscope

The main part of cathode ray oscilloscope is cathode ray tube which is also known as the heart of cathode ray oscilloscope.

The construction of cathode ray tube in order to understand the construction of cathode ray oscilloscope. Basically the cathode ray tube consists of five main parts and these main parts are written below:

1. Electron gun.
2. Deflection plate system.
3. Fluorescent screen.
4. Glass envelope.
5. Base.



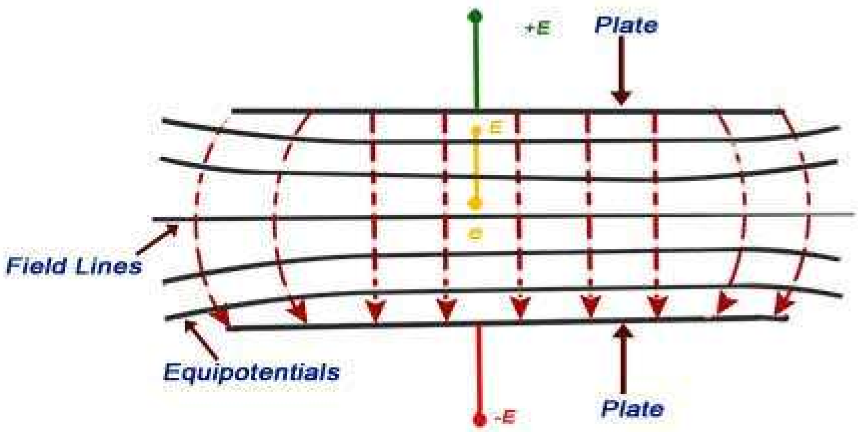
Electron Gun: It is the source of accelerated, energized and focused beam of electrons. It consists of six parts namely heater, a cathode, a grid, a pre-accelerating anode, a focusing anode and an accelerating anode. In order to obtain the high emission of electrons the layer of barium oxide (which is deposited on the end of cathode) is indirectly heated at moderate temperature. The electrons after this passes through a small hole called control grid which is made up of nickel. As the name suggests the control grid with its negative bias, controls the number of electrons or indirectly we can say the intensity of emitted electrons from cathode. After passing through the control grid these electrons are accelerated with the help of pre-accelerating and accelerating anodes. The pre-accelerating and accelerating anodes are connected to a common positive potential of 1500 volts.

Now after this the function of the focusing anode is to focus the beam of the electrons so produced. The focusing anode is connected to adjustable voltage 500 volts. Now there are two methods of focusing the electron beam and are written below:

1. Electrostatic focusing.
2. Electromagnetic focusing.

Here we will discuss electrostatic focusing method in detail.

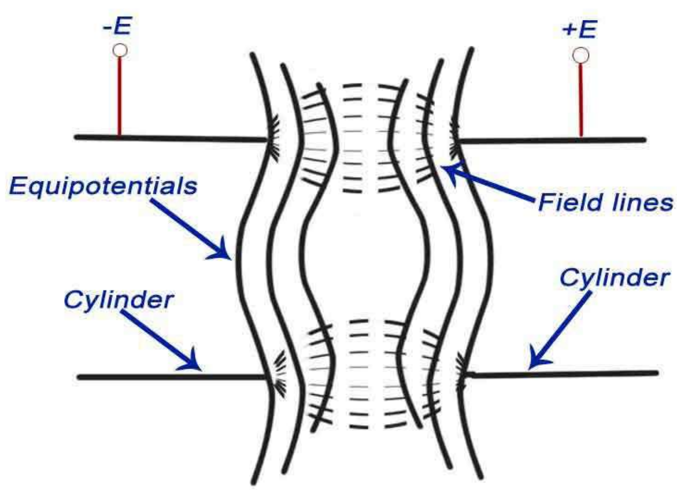
Electrostatic Focusing We know that the force on an electron is given by – qE, where q is the charge on electron ($q = 1.6 \times 10^{-19}$ C), E is the electric field intensity and negative sign shows that the direction of force is in opposite direction to that of electric field. Now we will this force to defect the beam of electrons coming out of electron gun. Let us consider two cases: Case One In this case we are having two plates A and B as shown in the figure.



Electric field between parallel Plates

The plate A is at potential +E while the plate B is at potential –E. The direction of electric field is from A plate to plate B at right angle to the surfaces of the plate. The equipotential surfaces are also shown in the diagram which is perpendicular to the direction of electric field. As the beam of electron passes through this plate system, it deflects in the opposite direction of electric field. The deflection angle can be easily varied by changing the potential of the plates.

Case Second Here we have two concentric cylinders with a potential difference applied between them as shown in the figure

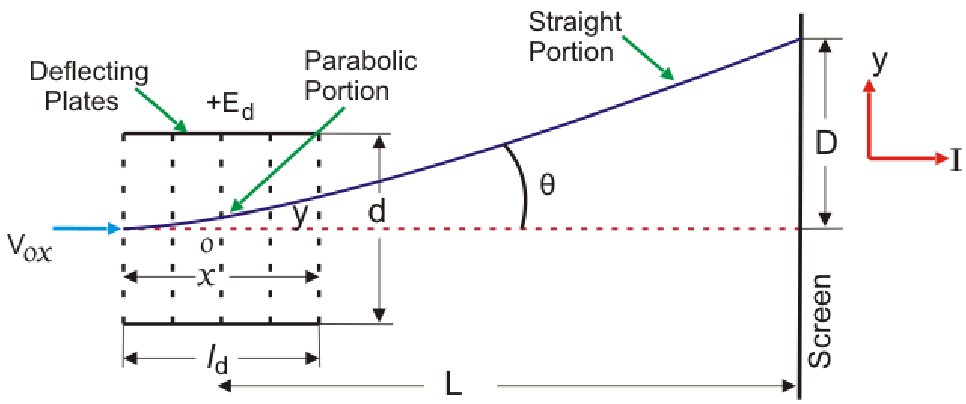


Field between two co-axial cylinders

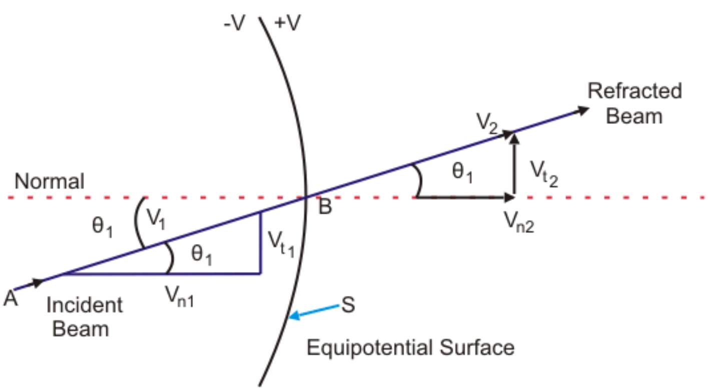
The resultant direction of electric field and the equipotential surfaces are also shown in the figure. The equipotential surfaces are marked by the dotted lines which are curved in shape. Now here we are interested in calculating the deflection angle of electron beam when it passes through this curved equipotential surface. Let us consider the curved equipotential surface S as shown below. The potential on the right side of the surface is +E while the potential on the left side of the surface –E. When a beam of electron is incident at angle A to the normal then it deflects by angle B after passing through the surface S as shown in the figure given below. The normal component of velocity of the beam will increase as force is acting in s direction normal to the surface. It means that the tangential velocities will remain same, so by equating the tangential components we have $V_1\sin(A) = V_2\sin(B)$, where V_1 is the initial velocity of the electrons, V_2 is the velocity after passing through the surface. Now we have relation as $\sin(A)/\sin(B) = V_2 / V_1$. We can from the above equation see that there is bending of the electron beam after passing through the equipotential surface. Therefore this system is also called focusing system.

Electrostatic Deflection

In order to find out the expression for the deflection, let us consider a system as shown below:



Electrostatic Deflection



Refraction of an Electron Beam

In the above system we have two plates A and B which are at potential +E and 0 respectively. These plates are also called deflection plates. The field produced by these plates is in the direction of positive y axis and there is no force along the x-axis. After deflection plates we have screen through which we can measure net deflection of the electron beam. Now let us consider a beam of electron coming along the x-axis as shown in the figure. The beam deflects by angle A, due presence of electric field and deflection is in the positive direction of y axis as shown in the figure. Now let us derive an expression for deflection of this beam. By the conservation of energy, we have loss in potential energy when the electron moves from cathode to accelerating anode should be equal to gain in kinetic energy of electron. Mathematically we can write,

$$eE = \frac{1}{2}mv^{\frac{1}{2}} \quad \cdots\cdots\cdots\cdots\cdots\cdots (1)$$

Where, e is the charge on electron, E is the potential difference between the two plates, m is the mass of electron, and v is the velocity of the electron. Thus, eE is loss in potential energy and $1/2mv^{1/2}$ is the gain in kinetic energy. From equation (1) we have velocity $v = (2eE/m)^{1/2}$. Now we have electric field intensity along the y axis is E/d, therefore force acting along the y axis is given by F = e E/d where d is the separation between the two deflection plates. Due to this force the electron will deflect along the y axis and let the deflection along y axis be equal to D which is marked on the screen as shown in the figure. Due to the force F there is net upward acceleration of the electron along positive y axis and this acceleration is given by Ee/(d × m).Since the initial velocity along positive y direction is zero therefore by equation of motion we can write the expression of displacement along y axis as,

$$y = \frac{1}{2}\left(\frac{Ee}{m \times d}\right) \times t^2 \quad \cdots\cdots\cdots\cdots\cdots\cdots (2)$$

As the velocity along the x direction is constant therefore we can write displacement as,

$$x = u \times t \cdots\cdots\cdots\cdots\cdots\cdots (3)$$

Where, u is velocity of electron along x axis. From equations 2 and 3 we have,

$$y = \frac{1}{2}\left(\frac{eE}{mu^2}\right) \times x^2 \quad \cdots\cdots\cdots\cdots\cdots\cdots (4)$$

This is the equation of trajectory of the electron. Now on differentiating the equation 4 we have slope

i.e. $$\frac{dy}{dx} = \frac{eEl}{mu^2}$$

Where, l is the length of the plate. Deflection on the screen can be calculated as,

$$D = L \times \frac{dy}{dx}$$

Distance L is shown in the above figure. Final expression of D can be written as,

$$D = \frac{LlE}{2dE}$$

From the expression of deflection, we calculate deflection sensitivity as,

$$\frac{D}{E} = \frac{Ll}{2dE}$$

Magnetic Deflection

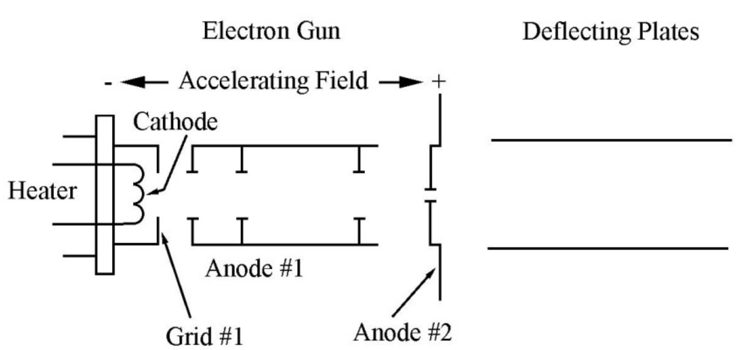
The Lorentz force law, Equation 1, tells us that a charged particle experiences a force in an area where there exists an electric or magnetic field.

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad (1)$$

where *F* is the force exerted on the charged particle, *q* is the charge of the particle, *E* is the electric field, *v* is the velocity of the charged particle and *B* is the magnetic field. When an electron (q = - e), is in a magnetic field, where $E = 0$, the electron experiences a force given by Equation 2.

$$\vec{F} = -e\left(\vec{v} \times \vec{B}\right) \qquad (2)$$

To examine the motion of an electron in a magnetic field, we will use a cathode ray tube. To examine these effects, we use a CRT. Enclosed within the cathode ray tube is an electron gun, Figure 1, which will be used to produce electrons with given energy.



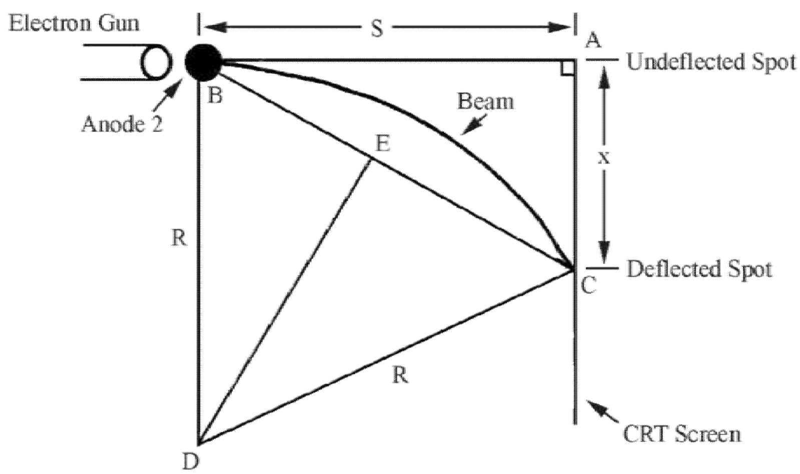
The electron gun will emit electrons with a kinetic energy equal to the charge of an electron times the accelerating voltage, V_{acc}. These electrons will be deflected by a magnetic field that is generated by a current-carrying wire. In this setup, the electron is moving perpendicular to the magnetic field. Thus, the force on the electron is:

$$F = evB \quad (3$$

Additionally, we know from last semester that, when the force acting on a body is perpendicular to the motion, the resulting motion is circular. Using this and conservation of energy ***SHOW*** that the electrons will move in a circle of radius *R*, given in equation 4:

$$R = \frac{m}{eB}\sqrt{\frac{2eV_{acc}}{m}} \quad (4)$$

The radius of curvature is related to the spot that is made on the screen, as seen in Figure 2. The electron beam is traveling from the left (where it is emitted by the electron gun) to the anode, and then on towards the screen. In the absence of a magnetic field this trajectory would be a straight line path, a distance *S* long; however, when a magnetic field is added (into the page) the electron beam will curve away from its original course, as seen in Figure 1



If the magnetic field is uniform, the force will be constant and the radius of curvature, *R*, is fixed. We can use geometry to determine how *R* is related to measurable quantities.

1. Δ BAC is similar to Δ DEB, since 2 sides are mutually perpendicular.

2. Using the Pythagorean theorem:

$$(\overline{BC})^2 = S^2 + x^2$$

3. For similar triangles, the ratio of sides is equal. Also, *DE* bisects *BC* . Thus:

$$\frac{x}{(\overline{BC})} = \frac{(\overline{BE})}{R} = \frac{\frac{1}{2}(\overline{BC})}{R}$$

3. From this, ***you*** should be able to ***SHOW***:

$$2Rx = (\overline{BC})^2$$

4. Using our result from line 2 and some algebra, ***you*** should be able to ***SHOW***:

$$R = \frac{S^2 + x^2}{2x} \approx \frac{S^2}{2x} \qquad (5)$$

where x is the deflection of the electron beam and ***S* = 0.213m** for the tubes used in this lab

Using equations 4 and 5, we find a preliminary expression for the deflection, x, in terms of the magnetic field strength:

$$\frac{S^2}{2x} \approx \frac{1}{B}\sqrt{\frac{2mV_{acc}}{e}} \qquad (6)$$

We cannot measure the strength of the magnetic field directly, but we can express it in terms of the current that produces it. To simplify the math, we will make another approximation. The extreme oblong rectangular geometry of the coils used to generate the magnetic field, B, means that the two "far ends" contribute relatively little. As such, the coil can be thought of as two sets of N long wires, where N is the number of turns in the coil. The magnetic field generated by a *single* long straight wire:

$$B = \frac{\mu_o I}{2\pi a} \qquad (7)$$

where a is the distance from the wire to the electron beam, I is the current which is generating B, and $\mu_0 = 4\pi \times 10^{-7}$ Tesla·m/Amp.

The magnetic field produced by the current in the top wire adds to the magnetic field produced by the current in the bottom wire. The wires on the top are about the same distance from the electron beam as the wires on the bottom. Since we are treating our coils as two sets of N wires, the magnetic field is:

$$B = B_{top} + B_{bottom} = N\left(\frac{\mu_o I}{2\pi a_{top}}\right) + N\left(\frac{\mu_o I}{2\pi a_{bottom}}\right) = \frac{\mu_o NI}{\pi a} \qquad (8)$$

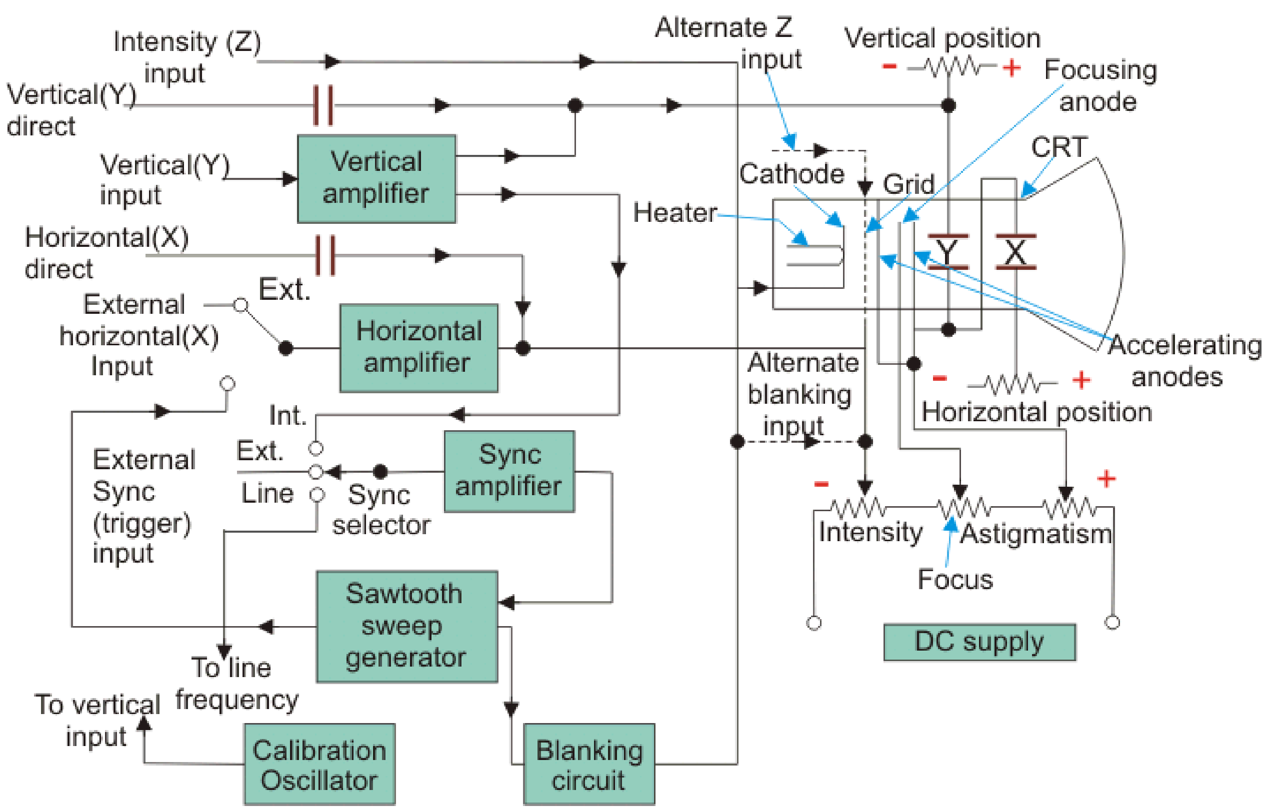
Substituting this result into Equation 6 and simplifying, you should be able to ***SHOW***:

$$x = \frac{\mu_o NS^2\sqrt{e/m}}{2\sqrt{2}\pi\, a}\frac{I}{\sqrt{V_{acc}}}$$

Graticule: These are the grid of lines whose function is to serve as a scale when the cathode ray oscilloscope is used for the amplitude measurements. There are three types of graticules and they are written below:

1. **Internal Graticule:** Internal graticule as name suggests deposited on the internal surface of the cathode ray tube face plate. There is no problem of parallax errors but we cannot change internal graticules as they are fixed.
2. **External graticule:**

Given below is the circuit diagram of cathode ray oscilloscope:



Basic Circuit Diagram of Cathode Ray Oscilloscope

Now we will study the basic circuit diagram of cathode ray oscilloscope under the following main parts.

1. **Vertical Deflection System:** The input signal for examining are fed to the vertical deflection system plates with the help of input attenuator and a number of amplifier stages. The main function of these amplifiers is to amplify the weak the weak signals so that the amplified signal can produce the desirable signals.
2. **Horizontal Deflection System:** Like the vertical system horizontal system also consists of horizontal amplifiers to amplify the weak input voltage signals but in contrast to vertical deflection system, horizontal deflection plates are fed by a sweep voltage that provides a time base as shown above. As shown in the circuit diagram, the saw tooth sweep generator is triggered by the synchronizing amplifier when the sweep selector switch is in the internal position and thus the triggered saw tooth generator gives input to the horizontal amplifier by following this mechanism. Now there are four types of sweeps:
    1. **Free Running or Recurrent Sweep** As the name suggests, the saw tooth waveform is repetitive i.e. a new sweep is started immediately after the previous sweep.
    2. **Triggered Sweep** Some time the waveform to be observed may not be periodic so it is desired that the sweep circuit remain inoperative and the sweep be initiated by the waveform under examination. In such cases we use triggered sweep.
    3. **Driven Sweep** Generally a driven sweep is used where the sweep is free running but triggered by the signal under test.
    4. **Non Saw Tooth Sweep** This is used for finding the phase difference between the two voltages. Another important application is that we can compare frequency of input voltages using non saw tooth sweep.
3. **Synchronization:** There must be synchronization between the sweep and the signal being measured. Synchronization is done to produce stationary pattern. There are three sources of synchronization which can be selected by synchronization selector and they are written below:
    1. **Internal** In this trigger is obtained from the signal being measured through vertical amplifier.
    2. **External** In this trigger an external trigger source is required.

3. **Line** In this method trigger is obtained power supply.
4. **Intensity Modulation** Intensity modulation can be done by inserting the signal between the ground and the cathode. Intensity modulation causes the brightening of the display.
5. **Positioning Controls** Position can be control by applying small independent internal direct voltage sources to the deflecting plates and with the help of potentiometer (using it as voltage divider) we can control the position of signal.
6. **Focus Control** Focus can be controlled by changing the focal length of the focusing electrode which acts like a lens and focal length can be changed by the changing potential of the focusing anode.
7. **Intensity Control** The intensity can be varied by changing the grid potential with respect to cathode.
8. **Calibration Circuit** Calibrating voltage has a square shape which is usually internally generated of known amplitude.
9. **Astigmatism** By adjusting the focus the spot can be made sharp in order to avoid the problem of astigmatism.

Intensity Modulation

This modulation is produced by inserting the signal between the ground and cathode.
This modulation causes by brightening the display.

Positioning Control

By applying the small independent internal direct voltage source to the detecting plates through the potentiometer the position can be controlled and also we can control the position of the signal.

Intensity Control

The intensity has a difference by changing the grid potential with respect to the cathode.

Applications of CRO

- Voltage measurement
- Current measurement
- Examination of waveform
- Measurement of phase and frequency

- Applications of CRO:

- **Measurement of voltage** – Voltage waveform will be made on the oscilloscope screen. From the screen of the CRO, the voltage can be measured by seeing its amplitude variation on the screen.
- **Measurement of current** – Current waveform will be read from the oscilloscope screen in the similar way as told in above point. The peak to peak, maximum current value can be measured from the screen.
- **Measurement of phase** – Phase measurement in cro can be done by the help of Lissajous pattern figures. Lissajous figures can tell us about the phase difference between two signals. Frequency can also be measured by this pattern figure.
- **Measurement of frequency** – Frequency measurement in cathode ray oscilloscope can be made with the help of measuring the time period of the signal to be measured.

Uses of CRO

In laboratory, the CRO can be used as

- It can display different types of waveforms
- It can measure short time interval
- In voltmeter, it can measure potential difference

Cathode Ray Oscilloscope Lissajous Patterns of CRO Measurement by Oscilloscope Digital Storage Oscilloscope Double Beam Oscilloscope Sampling Oscilloscope

Normally, an oscilloscope is an important tool in an electrical field which is used to display the graph of an electrical signal as it varies with respect to time. But some of the scopes have additional features apart from their fundamental use. Many oscilloscopes have the measurement tool that help us to measure waveform characteristics like frequency, voltage, amplitude, and many more features with accuracy. Generally, a scope can measure time-based as well as voltage-based characteristics.

Voltage Measurement

The oscilloscope is mainly voltage oriented device or we can say that it is a voltage measuring device. Voltage, current and resistance all are internally related to each other.

Just measure the voltage, rest of the values is obtained by calculation. Voltage is the amount of electric potential between two points in a circuit. It is measured from peak-to-peak amplitude which measures the absolute difference between the maximum point of signal and its minimum point of the signal. The scope displays exactly the maximum and minimum voltage of the signal received. After measuring all high and low voltage points, scope calculates the average of the minimum and maximum voltage. But you must be careful to mention which voltage you mean. Normally, oscilloscope has fixed input range, but this can be easily increased with the use of simple potential divider circuit.

Method to Measure Voltage

1. The simplest way to measure signal is to set the trigger button to auto that means oscilloscope start to measure the voltage signal by identifying the zero voltage point or peak voltage by itself. As any of these two points identified the oscilloscope triggers and measure the range of the voltage signal.
2. Vertical and horizontal controls are adjusted so that the displayed image of the sine wave is clear and stable. Now take measurements along the center vertical line which has the smallest divisions. Reading of the voltage signal will be given by vertical control.

Current Measurement

Electrical current cannot be measured directly by an oscilloscope. However, it could be measured indirectly within scope by attaching probes or resistors. Resistor measures the voltage across the points and then substituting the value of voltage and resistance in Ohm's law and calculates the value of electrical current. Another easy way to measure current is to use a clamp-on current probe with an oscilloscope.

Method to Measure Current

1. Attach a probe with the resistor to an electrical circuit. Make sure that resistor's power rating should be equal or greater than the power output of the system.
2. Now take the value of resistance and plug into Ohm's Law to calculate the current.

$$Current = \frac{voltage}{resistance}$$

According to Ohm's Law,

Frequency Measurement

Frequency can be measured on an oscilloscope by investigating the frequency spectrum of a signal on the screen and making a small calculation. Frequency is defined as the several times a cycle of an observed wave takes up in a second. The maximum frequency of a scope can measure may vary but it always in the 100's of MHz range. To check the performance of response of signals in a circuit, scope measures the rise and fall time of the wave.

Method to Measure Frequency

1. Increase the vertical sensitivity to get the clear picture of the wave on the screen without chopping any of its amplitude off.
2. Now adjust the sweep rate in such a way that screen displays a more than one but less than two complete cycles of the wave.
3. Now count the number of divisions of one complete cycle on the graticule from start to end.
4. Now take horizontal sweep rate and multiply it with the number of units that you counted for a cycle. It will give you the period of the wave. The period is the number of seconds each repeating waveform takes. With the help of period, you can simply calculate the frequency in cycles per second (Hertz).

Applications of CRO

In our previous post we discussed the block diagram of CRO and also studied various parts of it in detail. In this article we shall study some important applications of CRO , such as :

1. Examination of Waveform
2. Voltage measurement
3. Current measurement

1. Examination of Waveform

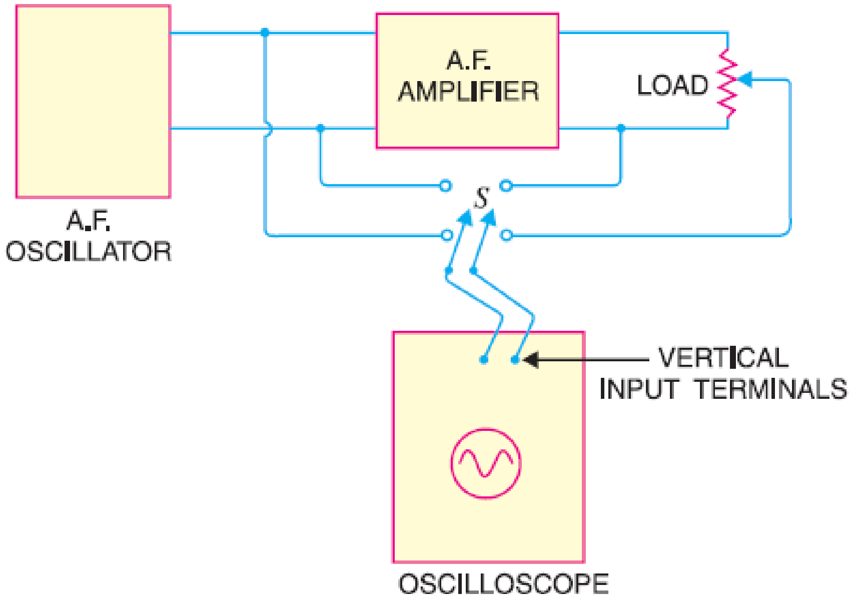
One of the important applications of CRO is to observe the wave shapes of voltages in various type of electronic circuits.

To do this, the signal under study is applied to vertical input terminals i.e. the ertical deflection plates of the oscilloscope.

The sweep circuit is set to internal so that saw tooth wave is applied to the horizontal input terminals i.e. the horizontal deflection plates.

Then various controls are adjusted to get sharp and well defined signal waveform on the screen.

Fig below shows the circuit for studying the performance of an audio frequency amplifier.



With the help of switch S, the output and input of amplifier is applied in turn to the vertical input terminals of the CRO. I f the waveforms are identical in shape, the fidelity of the amplifier is excellent.

Before discussing the next applications let us see how the signal waveform is displayed on CRO.

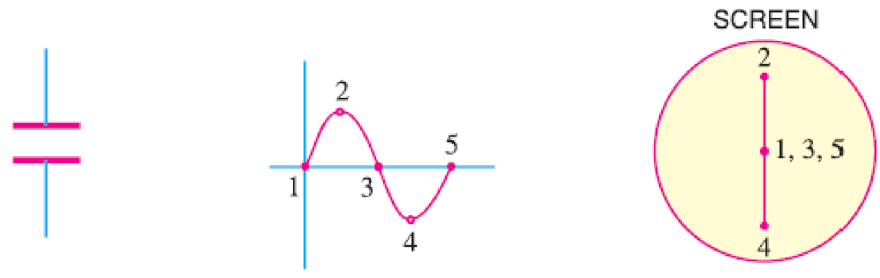
Display of signal waveforms on CRO

Applying Signal across Vertical Plates

When a sinusoidal voltage is applied to the vertical deflection plates, it makes the plates alternately positive and negative. Thus, in the positive half cycle, upper plate is positive and lower plate is negative and in the negative half cycle, the plate polarities are reversed.

As a result, the spot moves up and down at the same rate as the frequency of the applied voltage.

Since the frequency of the applied voltage is 50 Hz, hence we will see a continuous vertical line 2-1-4 on the screen as shown in fig below. This line gives no indication of the manner in which the voltage is alternating hence we cannot get the wave shape.



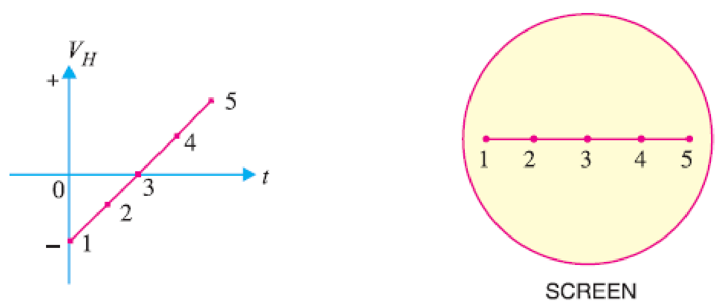
(i) (ii) (iii)

Applying Saw-tooth Wave across Horizontal Plates

To see the signal voltage variation with time on screen we have to move the beam horizontally from left to right at a uniform speed while it is moving up and down.

Again as soon as a full cycle of the signal is traced, the beam should return quickly to the left hand side of the screen so that it can start tracing the second cycle.

In order to move the beam from left to right at a uniform rate, a voltage that varies linearly with time should be applied to the horizontal plates. This can be achieved by applying a saw tooth wave as shown in fig below.



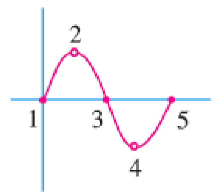
(i) (ii)

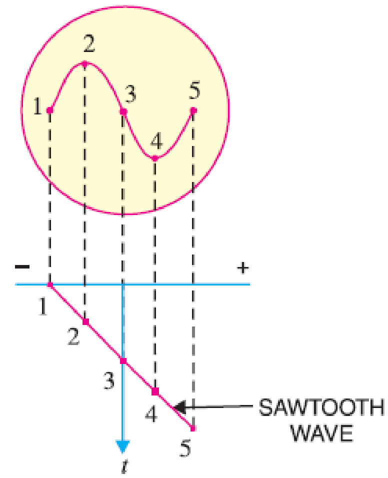
At time t=0, the negative voltage on the horizontal plate keep the beam to the extreme left on the screen as shown in the above figure .

As the time progresses, the negative voltage decreases linearly with time and the beam moves towards right forming a horizontal line. In this way, the saw tooth wave applied to horizontal plates moves the beam from left to right at a uniform rate.

Signal Pattern on Screen

When the signal voltage is applied to the vertical plates and saw-tooth wave to the horizontal plates, we get the exact pattern of the signal on the screen as shown in fig. below.





When the signal is at the instant 1, its amplitude is zero, and at this instant maximum negative voltage is applied to the horizontal plates. As a result the beam appears at extreme left on the screen as shown in fig.

When the signal is at instant 2, its amplitude is maximum, but the negative voltage on the horizontal plates is decreased at this instant. Hence, the beam is deflected upwards by the signal and towards right by the saw tooth wave. As a result, the beam strikes the screen at 2.

In a similar manner, the beam strikes the screen at 3,4 and 5 and we get the exact pattern of the signal on the screen.

2. Voltage Measurement

As we have already discussed, when the signal is applied to the vertical deflection plates only, a vertical line appears on the screen.

The height of the line is proportional to the peak-to-peak voltage of the applied signal.

To measure the voltage on CRO, the following steps are followed:

1. Shut off the internal horizontal sweep generator
2. Attach a tracing paper to the face of the oscilloscope. Mark off the paper with vertical and horizontal lines in the form of graph.
3. Now, calibrate the oscilloscope against a known voltage. Apply the known voltage to the vertical input terminals. Since, the sweep circuit is off, you will get a vertical line. Adjust the vertical gain till a good deflection is obtained. Let the deflection sensitivity is V volts/mm.
4. Keeping the vertical gain unchanged, apply the unknown voltage to be measured, to the vertical input terminals of the oscilloscope.

5. Measure the length of the vertical line obtained. Let it be l mm.
6. Now the unknown voltage = l×V volts.

3. Frequency Measurement

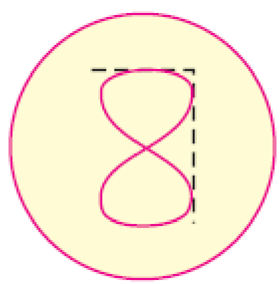
Using a CRO, the unknown frequency can be accurately determined following the below steps :

1. A known frequency is applied to the horizontal input and unknown frequency to the vertical input.
2. The various controls are adjusted.
3. A pattern with loops is obtained.
4. The number of loops cut by the horizontal line gives the frequency on the vertical plates (f_V) and the number of loops cut by the vertical line gives the frequency on the horizontal plates (f_H).

f_V / f_H = No. of loops cut by the horizontal line/No. of loops cut by the vertical line

To understand this better, let us take an example.

Suppose during the frequency measurement test, a pattern as shown in fig. below is obtained.



Let us assume that frequency applied to the horizontal plate is 2000 Hz.

Now, if we draw horizontal and vertical lines, we can see that one loop is cut by the horizontal line and two lops are cut by the vertical line.

Therefore,

$f_V / f_H = 1/2$

Or, $f_V / 2000 = 1/2$

Or, $f_V = 2000 \times 1/2 = 1000$ Hz

Hence unknown frequency is 1000 Hz.